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MIXED LAYER FORMULATION

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Mixed Layer Formulation

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1 Introduction

Ocean structure is modified by heat, fresh water, and momentum fluxes through the ocean surface. These directly impact on the top tens of metres of the water column. A further influence on the structure of the upper ocean is the small scale mixing which results, for example, from the dissipation of surface gravity waves. These factors combine to introduce a layer of vertically homogeneous water near the ocean surface. The thickness of this layer varies from only a few metres in regions of strong heating and little wind to over a hundred metres in the North Atlantic in winter. Evolution of sea surface temperature is influenced by the thickness of this layer.

A Kraus-Turner (1979) bulk mixed layer approach is taken in the Met. Office Unified Model. This calculates a balance between the energy available for mixing the water column and the introduction of buoyancy at the ocean surface. Water is mixed from the surface to a level at which no more energy is available for mixing.

This implementation is besed on the formulation of Gordon and Grahame (unpublished), which has been documented by Heathershaw and Martin (1987).
Because the mixed layer scheme is designed for use with the Met. Office ocean model its use of units is consistent with other aspects of the Unified Model system. Internal units within the mixed layer model are, therefore, **cgs** for compatibility with the ocean model. Forcing data, passed to the mixed layer model from outside the ocean model, are in **SI** units, but are converted within the mixed layer model before use. The scientific description in this paper is in cgs units.

This paper is divided into two main sections. Firstly, the scientific formulation of the model is given. This is followed by a commentary on the numerical implementation in the Unified Model.

2 Scientific description

Three major assumptions are made in the formulation of the mixed layer model:

all surface heat flux is used to heat (or cool) the column, all fresh water flux modifies the salinity, and all other tracer fluxes modify the corresponding tracer values;

all the kinetic energy available for mixing is used to increase the potential energy of the column;

mixing is instantaneous (ie on shorter timescales than the model timestep).

Because the model is based on principles of energy conservation, it is useful to define the heat content of the mixed layer (which is of depth d, and for convenience the vertical coordinate, z, is positive downwards):

$$H = \rho_0 c_p \int_0^d T(z)\ dz \qquad (1)$$

and the potential energy

$$P = -g \int_0^d \rho(z)\ z\ dz \qquad (2)$$

Consistent with the assumptions of the dynamics of the ocean model, ρ_0 is taken to be a constant reference density, T is the potential temperature of the water (expressed in °C), and c_p is the specific heat capacity of sea water at the surface (once again chosen to be constant over the whole ocean). The density used in the calculation of potential energy, $\rho(z)$, is taken to be the potential density with reference to the surface since the mixed layer scheme is designed to mix water from the surface downwards. These definitions are in accordance with normal conventions, namely that potential energy increases with distance from the centre of the Earth, and that the heat content is positive.

In the model, (1) and (2) must be discretized. Taking the boundaries between model layers to be h_n (where for convenience $h_0 = 0$), the heat content and potential energy of the mixed layer become:

$$H = \rho_0 c_p \sum_{j=1}^{N} (h_j - h_{j-1})\ T_j + \rho_0 c_p (d - h_N)\ T_d \qquad (3)$$

and

$$P = -g \sum_{j=1}^{N} (h_j^2 - h_{j-1}^2) \rho_j \ -g(d^2 - h_N^2) \rho_d \qquad (4)$$

It should be remembered that, as illustrated in fig 1, the bottom of the mixed layer calculated by the energy balance need not coincide with an interface between model layers. Care must be taken in redistributing energy in the bottom layer reached by the mixing to ensure conservation.

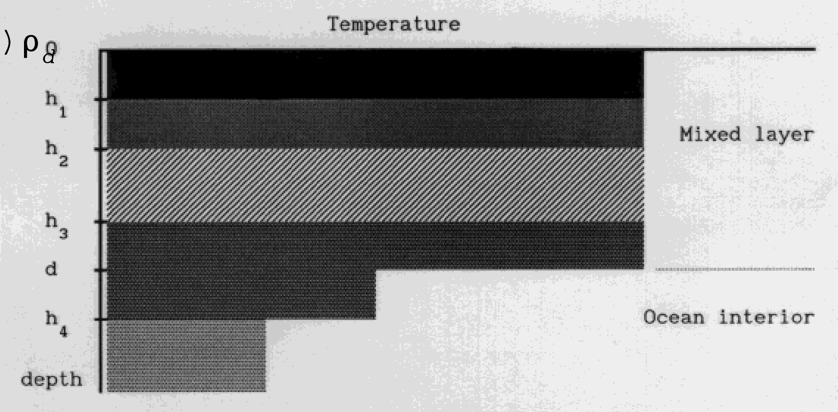

Figure 1: Relationship between bottom of mixed layer and model layers.

The basic quantities participating in the energy balance are:

Q_N	the non-penetrative heat flux (which contributes to the heating of the top layer;
$\Delta\rho_N$	the change in density at the surface due to non-penetrative heating and fresh water exchange with the atmosphere;
$Q_S(z)$	The penetrative (solar) component of the surface heat flux
$\Delta\rho_S(z)$	the density change due to the penetrative heat flux;
M_0	the "wind mixing" (mechanical) energy which is available for mixing water in a stably stratified column;
λ	the proportion of surface mechanical mixing which is available for mixing the ocean interior
δ	the rate of decay of mechanical energy with depth (ie $M(z) = M_0 \exp\{z/\delta\}$) ;
ϵ	the proportion of potential energy released by overturining of unstable layers which is available for mixing lower in the water column (thus allowing convection to penetrate into stable layers).

The mixed layer model may be divided into several stages:

(i) increment tracers with profile derived from surface fluxes and deplete mechanical energy

(ii) determine density changes arising from changed tracers profiles, and calculate the energy required to mix each layer internally (ie the work done to increase its potential energy)

(iii) working downwards from the surface, examine each layer in turn and mix tracers between the mixed layer (with uniform tracer values) and the new layer if (a) the two layers are unstable or (b) there is mechanical energy available to mix between stable layers.

Each of these stages is discussed below. Following the description of the stages, there is a discussion of modifications which are required if the ocean model uses variable timesteps with depth (ie distorted physics). In the description of the stages the symbol := is used to denote that the value of the symbol on the left is to be replaced with the value of the expression on the right, as in the FORTRAN assignment statement.

Stage (i): increment tracer values

This is straightforward, and is performed by another section of the ocean model code. Temperature changes in each layer are calculated by:

$$\Delta T = \Delta Q(z)\ \Delta t / (\Delta z c_p \rho_0)$$

where $\Delta Q(z)$ is the heat flux absorbed in the layer of thickness Δz , and Δt is the timestep. Similar calculations are performed for salinity and other tracers.

Depletion of the mechanical energy consists of multiplying the incoming energy (M_0) by the scaling factor (λ):

$$M := \lambda M_0 \tag{5}$$

Stage (ii) Calculate density and potential energy changes

For the idealised case (density a linear function of temperature and salinity) there is little complication involved in this. For the realistic case, in which there is a non-linear dependence of density on tracers values, it is necessary to make approximations.

It is straightforward to determine the density of the surface water before and after the non-penetrative and penetrative fluxes have been used to modify the tracers. This is done at the surface.

Assuming that the non-penetrative fluxes are absorbed at the ocean surface (a realistic assumption since the top model layer is of order metres thick and the fluxes are absorbed in a layer of a few centimetres), energy will be required to mix the layer (if the fluxes make the water less dense), or will be released by convection (if the water becomes denser). The potential energy change induced by mixing within the top layer is

$$\Delta P_N = -g \int_0^{h_1} z \, \Delta\rho \, dz = -\frac{1}{2} g \, h_1^2 \, \Delta\rho_N \tag{6}$$

(ie the potential energy added to the layer). If there is stabilisation ($\Delta\rho_N < 0$) the mixing energy is depleted, and it is increased if convection occurs.

$$M := M - \Delta P_N \tag{7}$$

(note that in this special case the potential energy released if the layer is cooled at the surface is not depleted before modifying the available mechanical energy, unlike when convection occurs between layers).

Below the surface, density is changed by the penetrative (solar) heat flux. The vertical profile of the heat flux is known a *priori*. This cannot be converted directly to a profile of density changes because of the nonlinear dependence of density on pressure, temperature and salinity.

Also in the mixed layer model a simplification is made in the determination of potential energy change with depth for the solar heat flux.

The profile of solar heating is assumed to be $Q_S(z)$, and the density change at layer n is determined by the change in temperature of the layer:

$$\frac{dT}{dt} = \frac{1}{c_p \rho_0} \frac{dQ_S}{dz} \tag{8}$$

or, in discrete form,

$$\Delta T_N = \Delta t \ (Q_S(H_{n-1}) - Q_S(h_n))/(\rho_0 c_p \Delta z_n) \tag{9}$$

Within each layer, more heat is absorbed in the top of the layer than in the bottom. Energy must be used to mix within the layers. Solar heating results in a stabilisation of the profile within the layer, adding potential energy equivalent to

$$\Delta P_S^1 = -g \int_{h_{n-1}}^{h_n} \Delta\rho(z) \, z \, dz \tag{10a}$$

although the potential energy in the mixed state is

$$\Delta P_m = -g\,\Delta\rho_n\,(h_n^2 - h_{n-1}^2) \qquad (10b)$$

so that the energy used to mix the layer is

$$\Delta P_S = \Delta P_S^1 - \Delta P_m \qquad (10c)$$

This energy is required whenever there is surface heating. It must be removed from the mechnical mixing which is available for mixing the column before proceeding to mix between layers:

$$M = M - \Delta P_S \qquad (11)$$

The full expression for density is highly non-linear in its dependence on temperature, salinity, and pressure. Although it would be possible to use the full expression to determine the density change in each layer due to the penetrative heating, this would be expensive. Instead a linearisation is used. Density is assumed to depend linearly on temperature, and vertical variations in the coefficient of proportionality are ignored. Horizontal variations are retained by calculating the density change at the surface using the full density calculation. Thus, at each level the density change due to the penetrative heat flux is in proportion to the surface density change, the coefficient of proportionality being the ration of the temperature change at that layer to that of the surface layer,ie

$$\Delta\rho_n = \Delta\rho_1\,(\Delta T_n/\Delta T_1))$$

This strategy was taken for numerical convenience: the calculation of density is one of the most expensive operations in the ocean model.

Stage (iii) mix adjacent layers

Starting at the second layer (provided there is ocean at that depth), the following sequence of calculations is performed for layer n (for mixing between the mixed layer and layer n).

The mechanical mixing is decayed according to

$$M = M\exp\{-(h_n - h_{n-1})/\delta\} \qquad (12)$$

Following this, the energy required to mix the water in the lower layer with that in the fully mixed mixed layer is calculated (ignoring nonlinear dependency of density on temperature and salinity):

$$E_{mix} = \begin{cases} \textit{potential energy intitial mixed layer + layer n} \\ -\ \textit{potential energy final state} \end{cases}$$

ie

$$\begin{aligned} 2E_{mix} = &-g\,\rho_{mix}\,h_{n-1}^2 - g\rho_n\,(h_n^2 - h_{n-1}^2) \\ &+ g\,\rho_{new}\,h_n^2 \end{aligned} \qquad (13)$$

and

$$\rho_{new} = \left(\rho_{mix}h_{n-1} + \rho_n(h_n - h_{n-1})\right)/h_n \qquad (14)$$

is the density arising from complete mixing of water in the mixed layer with the layer below.

A decision must be made as to whether mixing is possible.

If the energy required for mixing (E_{mix}) is greater than the mechanical energy available for mixing to level n (M) , partial mixing must be performed. The scheme used in this case is described later. The simpler case, where full mixing is possible, is described here.

Full mixing

If there is more mechanical mixing energy available than is needed to fully mix the water in the mixed layer with that in the layer below, all tracer values (Q) are set in the new mixed layer (which extends from the surface to layer n):

$$\Theta_{new} = (\Theta_{mix} h_{n-1} + \Theta_n (h_n - h_{n-1}))/h_n \tag{15}$$

and the new mixed layer values for tracers are set to Θ_{new} . In addition, the mixed layer density is set to ρ_{new} and the mixing energy modified. For the case of an initially stable stratification the energy required for mixing is subtracted from the available mechanical energy, while if the layers were convectivcely unstable, the mechanical energy is increased by a proportion of the potential energy released by convection:

$$M = M - \gamma E_{mix} \tag{16}$$

where $\gamma = 1$ (stable profile) or $\gamma = \epsilon$ (convection).

In cases of complete mixing, the calculated mixed layer depth is taken to be the base of the layer to which mixing has taken place (h_n).

Partial mixing

At the base of the mixed layer it is probable that there will insufficient mixing to entrain a complete layer into the mixed layer. This is accounted for by allowing partial mixing between layers in a manner which conserves tracers and energy. It should be remembered that this is a representation of a scale smaller than that modelled explicitly, and therefore the details of the incursion of the mixed layer into the lower layer will not be as accurate as if a higher resolution model were used. Nevertheless, during periods of mixed layer deepening, the time taken to fully entrain the lower layer is the same as if the process were calculated exactly (ie the energy input required for the mixing is the same in the finite difference and continuous equations). Detailed examination of the evolution of diagnosed mixed layer depth would, however, disclose an non-uniform increase in mixed layer depth with time in cases where the continuous equations would entrain at a constant rate.

Partial mixing occurs when $E_{mix} < M$. In this case, enough water is entrained from the layer below the mixed layer to use all the available energy to produce an increase the potential energy of the water column.

The heat content of the water column to the base of layer n is:

$$H = \rho_0 c_p (T_{mix} h_{n-1} + T_n \{h_n - h_{n-1}\}) \tag{17}$$

and after mixing the heat content is

$$H^1 = \rho_0 c_p (T^1_{mix} \{h_{n-1}\} + T^1_n \{h_n - h_{n-1}\}) \tag{18}$$

and conservation of heat implies that $H^1 = H$.

Before mixing the potential energy of the column is

$$P = -\frac{1}{2} g\, \rho_{mix} h^2_{n-1} - \frac{1}{2} g\, \rho_n (h^2_n - h^2_{n-1}) \tag{19}$$

and after mixing it is

$$P^1 = -\frac{1}{2} g\, \rho^1_{mix} (h^2_{n-1} - \frac{1}{2} g\, \rho^1_n (h^2_n - h^2_{n-1}) \tag{20}$$

where $P^1 = P + E_{mix}$

and

$$\Theta^1_{mix} = \{(1-b)\Theta_{mix} + b\,\Theta_n\} \quad (21)$$

$$\Theta^1_n = \{a\Theta_{mix} + (1-a)\Theta_n\} \quad (22)$$

for a tracer (or density) Θ .

These equations may be solved to give value for a and b. Conservation of heat (17, 18) yields an expression for b in terms of a (the derivation assumes $T_{n-1} \neq T_n$ which is always the case):

$$b = a\,\frac{(h_n - h_{n-1})}{h_{n-1}} \quad (23)$$

and conservation of potential energy closes the problem, yielding an expression for a (after much algebra):

$$a = b\,\frac{h_{n-1}}{h_n}\,\frac{M}{E_{mix}} \quad (24)$$

Although the mixed layer depth does not enter the calculations of energy balance, it is possible to define a level to which mixing would have occurred given a continuous model with density and tracer profiles identical to the discrete model. This diagnostic calculation assumes that heat and potential energy are conserved, and that mixing occurs to a depth $h_n + d$, which lies between layers in the finite difference model. Conservation of heat (mass and other tracers) yields:

$$\Theta_{new} = \frac{h_{n-1}\Theta_{mix} + d\,\Theta_n}{h_{n-1} + d} \quad (25)$$

and conservation of potential energy gives:

$$M = P_{new} - P_{old}$$

where

$$P_{new} = -\frac{1}{2}\,g\,\rho_{new}(h_{n-1} + d)^2$$

and

$$P_{old} = -\frac{1}{2}\,g\,\rho_{mix}h^2_{n-1} - g\,\rho_n(h_{n-1} + \frac{1}{2}d)\,d$$

which can be solved for d to give

$$d = \frac{2M}{g\,h_{n-1}(\rho_{new} - \rho_{mix})} + h_{n-1} \quad (26)$$

This quantity is useful for diagnostic calculations.

Modifications for distorted physics

Long spin-up integrations of the ocean model often use a technique known as "variable timestep with depth". This is equivalent to distorting the specific heat capacity of the differing layers in the water column. An alternative way of interpreting the distortion is that the timestep is truly different in each layer, but that instead of conserving *tracers* in vertical transfers between layers, the quantity to be conserved is the *tracer flux*. A constraint on the application of distorted physics is that the timestep should be uniform in regions affected by rapid changes, such as the mixed layer. In the model, however, it is not possible to guarantee that the mixed layer will be confined to a small number of layers, and so the mixed layer scheme must be modified to take account of the varying timesteps.

This is only done for the active parts of the mixed layer scheme. Mixed layer depth, which is a pure diagnostic, is not modified.

The expressions for the modified (new) values for tracers (during complete mixing) are given by:

$$\Theta_{new} = \frac{\rho_{mix}\upsilon_{n-1} + hro_n(\upsilon_n - \upsilon_{n-1})}{\upsilon_n} \qquad (27)$$

where $\upsilon = \int_0^z \gamma(z)\, z\, dz$ (usually γ decreases with depth so that $\upsilon_n < h_n$).

A similar scaling for mass (ie weighting by the effective timestep) allows (13) to be used in the following modified form in the case of variable timestep with depth.

$$\begin{aligned} 2E_{mix} = {} & -g\,\rho_{mix}\upsilon_{n-1}h_{n-1} - g\,\rho_n(\upsilon_n - \upsilon_{n-1})(h_n + h_{n-1}) \\ & + g\,\rho_{new}\upsilon_n h_n \end{aligned} \qquad (28)$$

When partial mixing occurs, the heat content of the water column to the base of layer n is:

$$H = \rho_0 c_p(T_{mix}\upsilon_{n-1} + T_n\{\upsilon_n - \upsilon_{n-1}\}) \qquad (29)$$

and after mixing the heat content is

$$H^1 = \rho_0 c_p(T^1_{mix}\{\upsilon_{n-1}\} + T^1_n\{\upsilon_n - \upsilon_{n-1}\}) \qquad (30)$$

and conservation of heat implies that $H^1 = H$.

Algebra using (21, 22, 29, 30) gives

$$b = a\,\frac{\upsilon_n - \upsilon_{n-1}}{\upsilon_{n-1}} \qquad (31)$$

and substituting into

$$P = -\frac{1}{2}g\,\rho_{mix}\,\upsilon_{n-1}h_{n-1} - \frac{1}{2}g\,\rho_n(h_n + h_{n-1})(\upsilon_n - \upsilon_{n-1}) \qquad (32a)$$

$$P^1 = -\frac{1}{2}g\,\rho^1_{mix}h_{n-1}\upsilon_{n-1} - \frac{1}{2}g\,\rho^1_n(\upsilon_n - \upsilon_{n-1})(h_n + h_{n-1}) \qquad (32b)$$

which are derived from (19, 20) and where $P^1 = P + M$

gives an expression for a:

$$a = \frac{2M}{g(\rho_n - \rho_{mix})(\upsilon_n - \upsilon_{n-1})h_n} \qquad (33)$$

or, rearranging

$$a = \frac{M\,\upsilon_{n-1}(h_n - h_{n-1})}{\left(E_{mix} + \frac{g}{2}\,\rho_n\{h_{n-1}\upsilon_n - h_n\upsilon_{n-1}\}\right)h_n(\upsilon_n - \upsilon_{n-1})} \qquad (34)$$

which closes the system and enables the values of tracers and density to be calculated in the partial mixing case.

3 Programming aspects

The structure of the mixed layer code generally follows the scientific description. Accordingly, this documentation only describes those aspects of the structure of the code which cannot be derived readily from the description in section 2.

There are two subroutines associated with the mixed layer model: MIXSET and MIXLAY. In addition, a modification is required to the subroutine STATE to add a further entry point for the calculation of a single layer of densities at the ocean surface. Cox option X is used to select the mixed layer code, for which the solar heating code is a prerequisite (option E).

Subroutine MIXSET

This calculates the constants required by the mixed layer model. These are: the scaling between density change at the surface and the change in potential energy required to mix the upper layer as a result of the non-penetrative fluxes (6); the scaling of surface density change to determine the potential energy required to mix penetrative flux - induced density changes at all layers throughout the water column (the vertical sum of (10, 11)); and the vertical profile of the decay of wind mixing energy to avoid calculating an exponential repeatedly. (12).

Subroutine MIXLAY

The major deviation of this code from the documentation is that the tracer values are not updated as a result of the surface fluxes by the mixed layer itself. It is assumed that this is done by other subroutines. Instead, the mixed layer model requires as input the surface density changes resulting from the penetrative and non-penetrative fluxes independently.

Other complications arise from the treatment of the special case when the timestep varies with depth. Instead of writing two separate versions of the subroutine, the code assumes that the timestep always varies with depth. This is achieved by taking the vertical grid spacing through two arguments: one the geometric layer thicknesses, and the other the scaled thicknesses. When running the model with uniform timesteps it is only necessary to call MIXLAY with both arguments set to the geometric thicknesses, and the code will work correctly.

As expressed in the scientific description,. it might be thought that the tracer values must be reset for all layers above that being entrained as each new layer is considered. This is not necessary, and the computational work is reduced by introducing a pointer for each horizontal position which records the lowest layer which has been entrained fully into the mixed layer. After all calculations have been completed, the mixed layer values are copied into those layers which the pointer indicates are within the mixed layer.

The final point which is worthy of note is that MIXLAY has the vertical index as its outermost loop. This means that it is possible to vectorise the calculations. Further economy is achieved by confining calculations to active points only.

Parameters used by the mixed layer model

There are three physical parameters which are used by the mixed layer model internally, the choice of which is a matter of judgement. These are set to the values

$$\epsilon = 0.15$$

$$\lambda = 0.7$$

and $\delta = 10000 cm\ (100 m)$

Calling sequences

```
SUBROUTINE MIXLAY (TA, WME,
+              IMT, KM, NT,
+              C2DTTS, GRAV,
+              DZ, ZDZ, RZ,
```

```
     +             FM, KMT,
     +             LAMBDA, EPSILON,
     +             DELPSF, DELPSL, DECAY,
     +             DRHO_SOL, DRHO_NET,
     +             DRHO_ICE,                          (@)
     +             DIAG_MLD
     +             )
```

Dimensioning arguments

```
       INTEGER
     +  IMT                    ! IN  Number of points in first dimension of arrays
     +, KM                     ! IN  Number of points in vertical
     +, NT                     ! IN  Number of tracers
```

Arguments which define model domain

```
       INTEGER
     +  KMT (IMT)              ! IN  Number of gridpoints in column
```

Physical arguments

```
       REAL
     +  C2DTTS                 ! IN  Timestep
     +, TA (IMT,KM,NT)         ! IN  Array of tracers
     +, WME (IMT)              ! IN  Wind mixing energy
     +, DZ (KM)                ! IN  Model layer thicknesses
     +, ZDZ (KM)               ! IN  Depth of base of layers
     +, RZ (KM)                ! IN  Weighted layer thicknesses
     +, FM (IMT, KM)   ! IN  Land/sea mask: 1.0=sea
     +, DELPSF                 ! IN  Non solar change density
     +, DELPSL (0:KM)          ! IN  Solar change in density
     +, DECAY (KM)             ! IN  Decay rate of WME with depth
     +, DRHO_SOL (IMT)         ! IN  Density change at surface: solar
     +, DRHO_NET (IMT)         ! IN  Density change at surface: net heat
     +, DRHO_ICE (IMT)         ! IN  Density change due to ice (@)
     +, DIAG_MLD (IMT)         ! OUT Instantaneous mixed layer depth
     +, GRAV                   ! IN  Acceleration due to gravity (cgs)
```

Note: Items marked with (@) are only specified when the sea ice model is present. The argument is omitted if there is no sea ice model, and when ice is present the value of this array is added to DRHO_NET.

```
       SUBROUTINE MIXSET      (DELPSF, DELPSL, DECAY,
     +                         GRAV, DZ, ZDZ, ZDZZ, RZ,
     +                         KM,
     +                 KFIX, RSOL, ETA1, ETA2, DELTA
     +                         )
```

Dimensioning arguments

```
         INTEGER
     +          KM             ! IN  Number of points in vertical
```

Mapping of physical scales to model layers

```
         INTEGER
     +  KFIX           ! IN  Number of levels with solar heating
```

Physical arguments

```
REAL
+   GRAV                 ! IN  Acceleration due to gravity (cgs)
+,  DZ (KM)              ! IN  Layer thicknesses
+,  ZDZ (KM)             ! IN  Layer bottoms
+,  ZDZZ (KM)  ! IN  Layer centres
+,  RZ (KM)              ! IN  Thickness scaled by 1/timestep ratio
+,  RSOL                 ! IN  Ratio of solar components
+,  ETA1                 ! IN  Decay scale (cgs) of one solar component
+,  ETA2                 ! IN  Decay scale (cgs) of other solar component
+,  DECAY (KM)           ! OUT Decay scale for wind mixing energy
+,  DELPSF               ! OUT Energy change due to surface
+,  DELPSL (0:KM)        ! OUT Energy change from solar
```

References

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