P244 The surface and boundary layer equations and their implicit solution

(i) Introduction

The surface and boundary layer scheme updates the deep soil temperatures (for land points), the grid-box mean surface temperature (for land points and sea points with sea-ice), the liquid/frozen water temperatures, total water contents and horizontal wind components for the lowest BL_LEVELS atmospheric layers.

Surface moisture fluxes are output for use in the surface and subsurface hydrology component (P25) which updates the amount of lying snow, the surface (or "canopy") water store and the soil moisture. The prognostic equations for the deep soil temperatures and the numerical scheme for calculating their increments are described in the documentation for the soil thermodynamics subcomponent (P242).

(ii) The surface temperature equation

For sea points with no sea-ice the surface temperature is predicted by an ocean model, if a coupled configuration is used, or prescribed from a climatology or from an analysis of observations for atmosphere only configurations. In either configuration the sea surface temperature is input but not updated by the surface and boundary layer scheme. For sea-ice and land points prognostic equations for the grid-box mean surface temperature have to be solved.

For sea points with sea-ice, i.e. $f_I > 0$, the rate of change of the grid-box mean surface temperature is given by the prognostic equation

$$\frac{dT_*}{dt} = A_I \left(R_{N\downarrow *(I)} - H_I - H_{*(I)} - LE_{*(I)} \right) \qquad \text{(P244.1)}$$

where A_I is the reciprocal effective areal heat capacity of sea-ice (set to a uniform value of $4.8x10^{-6}\ j^{-1}\ m^2\ K$ in the model). $R_{N\downarrow *(I)}$ is the net ownward radiative flux at the sea-ice surface (weighted with the sea-ice fraction to give a gridbox mean value) input from the radiation scheme. H_I is the grid-box mean heat flux through the sea-ice (positive downwards) calculated in terms of timelevel n quantities using (P241.3) in the sea-ice thermodynamics subroutine. $H_{*(I)}$ and $E_{*(I)}$ are the (weighted) heat and moisture fluxes for the ice as given by (P2430.13) and (P2430.14). $L = L_C + L_F$ is the latent heat appropriate for sublimation of sea-ice.

For land points the rate of change of surface temperature is given by

$$\frac{dT_*}{dt} = A_{S1}\left(R_{N\downarrow *} - H_S - H_* - LE_*\right) \quad \text{(P244.2)}$$

where A_{S1} is the reciprocal areal heat capacity of the top soil layer defined by (P242.5) and calculated in the soil thermodynamics subroutine using (P242.11) and (P242.14). $R_{N\downarrow *}$ is the net downward radiative flux at the surface calculated in the radiation scheme. H_S is the heat flux from the top to next-to-top soil layer calculated in the soil thermodynamics subroutine in terms of timelevel n quantities using (P242.8) with $r = 1$. H_* and E_* are the sensible heat and moisture fluxes at the surface respectively (both positive upwards); the latent heat L is set to $L_C + L_F$ when there is lying snow and to L_C otherwise.

The finite difference forms of (P244.1) and (P244.2) incorporate an implicit treatment of the turbulent fluxes. Discretizing the sea-ice surface temperature equation, (P244.1), with respect to time gives the increment δT_* as

$$\delta T_* = (\delta T_*)_{RN} + (\delta T_*)_{IF} + (\delta T_*)_{TF} \quad \text{(P244.3)}$$

where the T_* increments due to the net radiative flux, the turbulent fluxes and the heat flux through the sea-ice are:

$$(\delta T_*)_{RN} = \delta t A_I R_{N\downarrow *(I)} \quad \text{(P244.4)}$$

$$(\delta T_*)_{TF} = -\delta t A_I (H_{*(I)} + LE_{*(I)}) \quad \text{(P244.5)}$$

$$(\delta T_*)_{IF} = -\delta t A_I H_I \quad \text{(P244.6)}$$

Similarly the time discretized of the land surface temperature equation, (P244.2), can be written as

$$\delta T_* = (\delta T_*)_{RN} + (\delta T_*)_{SF} + (\delta T_*)_{TF} \quad \text{(P244.7)}$$

where the increments due to the net radiative flux, the turbulent fluxes and the soil heat flux are respectively

$$(\delta T_*)_{RN} = \delta t A_{S1} R_{N\downarrow *} \qquad \text{(P244.8)}$$

$$(\delta T_*)_{TF} = -\delta t A_{S1}(H_* + LE_*) \qquad \text{(P244.9)}$$

$$(\delta T_*)_{SF} = -\delta t A_{S1} H_S \qquad \text{(P244.10)}$$

To complete the description of the time discretization H_* , E_* , $H_{*(I)}$ and $E_{*(I)}$ are

specified as

$$\begin{aligned} H_* &= -c_P\, \rho^n_* c^n_H v^n_{sh}\Big(\gamma_1 (T^{n+1}_{L1} - T^{n+1}_* + (1-\gamma_1)(T^n_{L1} - T^n_*) + g(z_1^n + z_{0m} - z_{0h})/c_P\Big) \\ &= -c_P\, \rho^n_* c^n_H v^n_{sh}\Big((T^n_{L1} - T^n_* + g(z_1^n + z_{0m} - z_{0h}/c_P) + \gamma_1(\delta T_{L1} - \delta T_*\Big) \\ &= H^n_* - \gamma_1 c_P R K_H(1)(\delta T_{L1} - \delta T_*) \end{aligned}$$

(P244.11)

$$\begin{aligned} E_* &= -\rho^n_* c^n_E v^n_{sh}\Big(\gamma_1 (\, q^{n+1}_{W1} - q_{SAT}(T^{n+1}_*, p^n_*) + (1-\gamma_1)(q^n_{W1} - q_{SAT}(T^n_*, p^n_*))\Big) \\ &= -\rho^n_* c^n_E v^n_{sh}\Big((q^n_{W1} - q_{SAT}(T^n_*, p^n_*)) + \gamma_1(\delta q_{W1} - \alpha^n_* \delta T_*)\Big) \\ &= E^n_* - \gamma_1 R K_E (\delta q_{W1} - \alpha^n_* \delta T_*) \end{aligned}$$

(P244.12)

$$\begin{aligned} H_{*(I)} &= -f_I c_P\, \rho^n_* c^n_H v^n_{SH}\Big(\gamma_1 (T^{n+1}_{L1} - T^{n+1}_{*(I)} + (1-\gamma_1)(T^n_{L1} - T^n_{*(I)}) + g z_1^n / c_P\Big) \\ &= -f_I c_P\, \rho^n_* c^n_H v^n_{sh}\Big((T^n_{L1} - T^n_{*(I)} + g z_1^n / c_P) + \gamma_1(\delta T_{L1} - \delta T_{*(I)})\Big) \\ &= H^n_{*(I)} - f_I \gamma_1\, c_P R K_H(1)(\delta T_{L1} - \delta T_{*(I)}) \\ &= H^n_{*(I)} - \gamma_1 c_P R K_H(1)(f_I \delta T_{L1} - \delta T_*) \end{aligned}$$

(P2440.1)

$$
\begin{aligned}
E_{*(I)} &= -f_I\, \rho_*^n c_E^n v_{sh}^n \Big(\gamma_1 (q_{W1}^{n+1} - q_{SAT}(T_{*(I)}^{n+1}, p_*^n)) \\
&\qquad + (1-\gamma_1)(q_{W1}^n - q_{SAT}(T_{*(I)}^n, P_*^n)) \Big) \\
&= -f_I \rho_*^n c_E^n v_{sh}^n \Big((q_{W1}^n - q_{SAT}(T_{*(I)}, p_*^n)) + \gamma_1 (\delta q_{W1} - \alpha_{*(I)} \delta T_{*(I)}) \Big) \\
&= E_{*(I)} - f_I \gamma_1 R K_E (\delta q_{W1} - \alpha_{*(I)}^n \delta T_{*(I)}) \\
&= E_{*(I)}^n - \gamma_1 R K_E (f_I \delta q_{W1} - \alpha_{*(I)}^n \delta T_*)
\end{aligned}
$$

(P2440.2)

An expansion of q_{SAT} to first order in δT_* about T_*^n has been applied in (P244.12) to approximate the function's value at T_*^{n+1} (and similarly to first order in $\delta T_{*(I)}$ in (P2440.2)):

$$q_{SAT}(T_*^{n+1}, p_*^n) \simeq q_{SAT}(T_*^n, p_*^n) + \alpha_*^n\, \delta T_* \qquad \text{(P244.13)}$$

The coefficient α_*^n introduced into (P244.12) by this approximation can be calculated using the Clausius-Clapeyron equation:

(P244.13a) where $L = L_C + L_F$ for sea-ice and snow covered land $\alpha_*^n = \left.\dfrac{\partial q_{SAT}}{\partial T_*}\right|_{T_*^n} = \dfrac{\epsilon L q_{SAT}(T_*^n, p_*^n)}{R(T_*^n)^2}$ but $L = L_C$ for ice-free sea and snow-free land. $\alpha_{*(I)}^n$ in (P2440.2) can be calculated using (P244.13a) with $T_{*(I)}$ (given by (P2430.1)) replacing T_* .

When $|\delta T_*|$ is large the use of (P244.13a) in the r.h.s. of (P244.13) can give a poor estimate of $q_{SAT}(T_*^{n+1}, p_n^*)$; indeed when δT_* is large and negative it can give a physically unrealistic negative value for q_{SAT} . For this reason the formulation for α_*^n as a local gradient at T_*^n is abandoned when $|(\delta T_*)_{ex}| > 2\ K$ (at sea-ice points when $|\delta T_{*(I)}| > 2\ K$ in favour of the following finite difference expression:

$$\alpha_*^n = \frac{q_{SAT}(T_*^n + (\delta T_*)_{ex}, p_*^n) - q_{SAT}(T_*^n, p_*^n)}{\delta T_*)_{ex}} \qquad \text{(P244.13b)}$$

The explicit increment to T_*, $(\delta T_*)_{ex}$, is given by (P244.15) or (P244.19) below. At sea-ice points $T_{*(I)}$ replaces T_* in (P244.13b) and $(\delta T_{*(I)})_{ex} = (\delta T_*)_{ex} / f_I$ should replace $(\delta T_*)_{ex}$.

The area weighted surface heat and moisture fluxes for the leads are specified as

$$\begin{aligned} H_{*(L)} &= -(1-f_I)\, c_P\, \rho_*^n c_H v_{sh}^n (\gamma_1 (T_{L1}^{n+1} - T_{FS}) \\ &\qquad + (1-\gamma_I)(T_{L1}^n - Y_{FS}) + g(z_1^n + Z_{0m(SEA)} - Z_{0k(SEA)}/c_P) \\ &= -(1-f_I)\, c_P\, \rho_*^n v_{sh}^n \Big((T_{L1}^n - T_{FS} + g(z_1^N + z_{0m(SEA)} - z_{0h(SEA)})/c_P + \gamma_1 \delta T_{L1} \Big) \\ &= H_{*(L)} - (1-f_I)\gamma_1 c_P R K_H(1)\, \delta T_{L1} \end{aligned}$$

(P2440.3)

$$\begin{aligned} E_{*(L)} &= -(1-f_I)\, \rho_*^n c_E^n v_{sh}^n (\gamma_1 (q_{W1}^{n+1} - q_{SAT}(T_{FS}^n, p_*)) \\ &\qquad + (1-\gamma_1)(q_{W1}^{n+1} - q_{SAT}(T_{FS}, p_*^n))) \\ &= -(1-f_I)\, \rho_*^n c_E^n v_{sh}^n \Big((q_{W1}^n - q_{SAT}(T_{FS}, p_*^n)) + \gamma_1 \delta q_{W1} \Big) \\ &= E_{*(L)}^n - (1-f_I)\gamma_1 R K_E \delta q_{W1} \end{aligned} \qquad \text{(P2440.4)}$$

The total fluxes at sea-ice points are found by adding the ice and leads components (see (P2430.15) and (P2430.16)); it can easily be seen that H_* and E_* are still given by (P244.11) and (P244.12).

The parameter γ_1 in the above flux specifications is the forward timelevel (i.e.timelevel n+1) weighting factor for the surface fluxes in the implicit numerical scheme. Note that $\gamma_1 = 0$ gives an explicit scheme, i.e. a scheme in which δT_* is given by (P244.3) and (P244.7) with only timelevel n quantities on the right hand sides. However, a non-zero value is used in the model as discussed below. The timelevel n quantities $H_{*(I)}$, $E_{*(I)}$, $RK_H(1)$ and $RK_E(1)$ are calculated in subcomponent P243 (see P243.134,135, P2430.21,22, P243.118-120, P2430.17-20, P243.125 and129).

Substituting for H_* and E_* from (P244.11) and (P244.12) into (P244.9), the equation for δT_* can be written as

$$A_{T*}\delta T_{L1} + B_{T*}\delta T_* + C_{T*}\delta q_{W1} = (\delta T_*)_{ex} \qquad \text{(P244.14)}$$

and at sea-ice points after substituting for $H_{*(I)}$ and $E_{*(I)}$ from (P2440.1) and (P2440.2) into (P244.5) as

$$f_I A_{*T}\delta T_{L1} + B_{T*}\delta T_* + \int_I C_{T*}\delta q_{W1} = (\delta T_*)_{ex} \qquad \text{(P2440.5)}$$

where the explicit T_* increment, T_*, $(\delta T_*)_{ex}$, and the coefficients A_{T*}, B_{T*} and C_{T*} are given below:

For land points:

$$(\delta T_*)_{ex} = \delta t A_{S1}(R_{N\downarrow *} - H_S - (H_*^n + LE_*^n)) \qquad \text{(P244.15)}$$

$$A_{T*} = -\gamma_1 \delta t A_{S1} RK_{(H)}(1) \qquad \text{(P244.16)}$$

$$C_{T*} = -\gamma_1 \delta t A_{S1} RK_E L \qquad \text{(P244.17)}$$

$$B_{T*} = 1 - A_{T*} - C_{T*}\alpha_*^n \qquad \text{(P244.18)}$$

$L = L_C$ for snow-free land and $L = L_C + L_F$ for snow covered land.

For sea points with sea-ice $(f_I > 0)$:

$$(\delta T_*)_{ex} = \delta t A_I (R_{N\downarrow(I)*} - H_I - (H^n_{*(I)} + LE^n_{*(I)})) \qquad \text{(P244.19)}$$

$$A_{T*} = -\gamma\delta A_I c_P R K_H(1) \qquad \text{(P244.20)}$$

$$C_{T*} = -\gamma\delta A_I R K_E(L) \qquad \text{(P244.21)}$$

$$B_{T*} = 1 - A_{T*} - C_{T*}\alpha^n_{*(I)} \qquad \text{(P244.22)}$$

$$L = L_C + L_F$$

Finally for sea points without sea-ice $(f_I = 0)$ (P2440.5) is still valid if:

$$(\delta T_*)_{ex} = 0, \quad A_{T*} = 0, \quad C_{T*} = 0, \quad B_{T*} = 0.$$

Scheme 1: Local mixing

(iii).1 The equation for turbulent mixing of heat

The prognostic equation for the liquid/frozen water temperature, T_L , is

$$\frac{\partial T_L}{\partial t} = g\frac{\partial F_{TL}}{\partial p} + \left.\frac{\partial T_L}{\partial t}\right|_{nt} \qquad \text{(P244.23)}$$

where $(\partial T_L/\partial t)\big|_{nt}$ is the rate of change of T_L from all the other parts of the model, i.e. the non-turbulent (nt) contributions. The finite difference form of (P244.23) for atmospheric layer 1 is

$$\delta T_{L1} = (g\delta t / \Delta p_1)(F_{TL}(2) - F_{TL}(1)) + (\delta T_{L1})_{nt} \qquad \text{(P244.24)}$$

where $F_{TL}(2)$ is the flux from layer 1 to 2 and $F_{TL}(1)$ is the flux from the surface into layer 1.

The implicit numerical integration scheme is defined by specifying these fluxes as:

$$F_{TL}(1) \equiv H_*/c_P - \gamma_1 RK_H(1)(\delta T_{L1} - \delta T_*) \qquad \text{(P244.25)}$$

(see (P244.11) above) and

$$\begin{aligned} F_{TL}(2) &= -RK_H(2)(\gamma_2(T^{n+1}_{L2} - T^{n+1}_{L1}))/\Delta z_{1+1/2} \\ &\qquad + (1-\gamma_2)(T^n_{L2} - T^n_{L1})/\Delta z_{1+1/2} + g/c_P) \\ &= F^n_{TL}(2) - \gamma_2 RK_H(2)(\delta T_{L2} - \delta T_{L1})/\Delta z_{1+1/2} \end{aligned}$$

(P244.26)

The timelevel n quantities H^n_*, $RK_H(1)$, $F^n_{TL}(2)$ and $RK_H(2)$ are calculated in subcomponent P243 (see (P243.134, 118, 125, 142, 145)). γ_2 is the forward timelevel weighting factor for fluxes between layers 1 and 2. The values of γ_k used in the model are discussed below.

Substituting for $F_{TL}(1)$ and $F_{TL}(2)$ from (P244.25) and (P244.26) in (P244.24) gives

$$A_{T1}\delta T_{L2} + B_{T1}\delta T_{L1} + C_{T1}\delta T_* = (\delta T_{L1})_{ex} + (\delta T_{L1})_{nt} \qquad \text{(P244.27)}$$

where the dimensionless coefficients A_{T1}, B_{T1} and C_{T1} are given by

$$A_{T1} = \gamma_2 (g\delta t/\Delta p_1) RK_H(2)/\Delta z_{1+1/2} \qquad \text{(P244.28)}$$

$$C_{T1} = \gamma_1 (g\delta t/\Delta p_1) RK_H(1) \qquad \text{(P244.29)}$$

$$B_{T1} = 1 - A_{T1} - C_{T1} \qquad \text{(P244.30)}$$

and the explicit increment to T_{L1} due to turbulent mixing is given by

$$(\delta T_{L1})_{ex} = (g\delta t/\Delta p_1)(F^n_{TL}(2) - H^n_*/c_P) \qquad \text{(P244.31)}$$

Extending the same implicit treatment to the layers above layer 1, the finite difference form of the equation for T_{LK} $(2 \leq k \leq BL\_LEVELS - 1)$ is

$$\delta T_{Lk} = (g\delta t/\Delta p_k)(F_{TL}(k+1) - F_{TL}(k)) + (\delta T_{Lk})_{nt} \quad \text{(P244.32)}$$

$F_{TL}(k)$ is the flux from layer $k - 1$ to layer k $(2 \leq k \leq BL\_LEVELS)$. The implicit numerical integration scheme is defined by specifying this flux as:

$$\begin{aligned} F_{TL}(k) &= -RK_H(k)(\gamma_k(T^{n+1}_{Lk} - T^{n+1}_{Lk-1})/\Delta z_{k-1/2} \\ &\qquad + (1-\gamma_k)(T^n_{Lk} - T^n_{Lk-1})/\Delta z_{k-1/2} + g/c_p) \quad \text{(P244.33)} \\ &= F^n_{TL}(k) - \gamma_k RK_H(k)(\delta T_{Lk} - \delta T_{Lk-1})/\Delta z_{k-1/2} \end{aligned}$$

γ_k is the forward timelevel weighting factor for fluxes between layers $k - 1$ and k. The timelevel n quantities $RK_H(k)$ and $F^n_{TL}(k)$ are calculated in subcomponent P243 (see (P243.142, 140, 145)). Substituting for the fluxes from (P244.33) in (P244.32) gives, for $2 \leq k \leq BL\_LEVELS - 1$

$$A_{Tk}\delta T_{Lk-1} + B_{Tk}\delta T_{Lk} + C_{Tk}\delta T_{Lk-1} = (\delta T_{Lk})_{ex} + (\delta T_{Lk})_{nt} \quad \text{(P244.34)}$$

where the dimensionless coefficients are given by

$$A_{Tk} = \gamma_{k+1}(g\delta t/\Delta p_k) RK_H(k+1)/\Delta z_{k+1/2} \quad \text{(P244.35)}$$

$$C_{Tk} = \gamma_k(g\delta t/\Delta p_k) RK_H(k)/\Delta z_{k-1/2} \quad \text{(P244.36)}$$

$$B_{Tk} = 1 - A_{Tk} - C_{Tk} \quad \text{(P244.37)}$$

and the explicit increment to T_{Lk} due to turbulent mixing is given by

$$(\delta T_{Lk})_{ex} = (g\delta t/\Delta p_k)\left(F^n_{TL}(k+1) - F^n_{TL}(k)\right) \quad \text{(P244.38)}$$

For $k = BL\_LEVELS$ the only difference to the treatment for the layers below is in the assumption that the flux at the top of layer BL_LEVELS is zero.

This gives the following equation in place of (P244.34):

$$B_{Tk}\delta T_{Lk} + C_{Tk}\delta T_{Lk-1} = (\delta T_{Lk})_{ex} + (\delta T_{Lk})_{nt} \quad \text{(P244.39)}$$

where

$$C_{Tk} = \gamma_k (g\delta t/\Delta p_k) RK_H(k) / \Delta z_{k-1/2} \quad \text{(P244.40)}$$

$$B_{Tk} = 1 - C_{Tk} \quad \text{(P244.41)}$$

and the explicit increment to T_{LK} due to turbulent mixing is given by

$$(\delta T_{LK})_{ex} = -(g\delta t/\Delta p_k) F^n_{TL}(k) \quad \text{(P244.42)}$$

where $k = BL\_LEVELS$ in (P244.39)-(P244.42).

(iv).1 The equation for turbulent mixing of moisture

The prognostic equation for the total water content, q_W is

$$\frac{\partial q_W}{\partial t} = g\frac{\partial F_{qW}}{\partial p} + \left.\frac{\partial q_W}{\partial t}\right|_{nt} \quad \text{(P244.43)}$$

The finite difference form of (P244.43) (for $1 \leq k \leq BL\_LEVELS$) follows by specifying the fluxes as

$$\begin{aligned} F_{qw}(k) &= -RK_H(k)(\gamma_k(q^{n+1}_{wK} - q^{n+1}_{Wk-1})/\Delta z_{k-1/2} \\ &\quad + (1-\gamma_k)(q^n_{Wk} - q^n_{Wk-1})/\Delta z_{k-1/2}) \quad \text{(P244.44)} \\ &= F^n_{qW}(k) - \gamma_k RK_H(k)(\delta q_{Wk} - \delta q_{Wk-1})/\Delta z_{k-1/2} \end{aligned}$$

for $2 \leq k \leq BL\_LEVELS$ and the surface moisture flux, $F_{qW}(1) = E_*$, as given by

(P244.12). The explicit flux of total water from layer $k - 1$ to layer k, $F^n_{qW}(k)$, is calculated

in subcomponent P243 using (P243.146). The increment to q_{W1} is given by

$$\alpha^n_* A_{Q1} \delta T_* + B_{Q1} \delta q_{W1} + C_{Q1} \delta q_{W2} = (\delta q_{W1})_{ex} + (\delta q_{W1})_{nt} \quad \text{(P244.45)}$$

where the dimensionless coefficients are given by

$$A_{Q1} = \gamma_1 (g \delta t / \Delta p_1) RK_E \quad \text{(P244.46)}$$

$$C_{Q1} = \gamma_2 (g \delta t / \Delta p_1) RK_H(2) / \Delta z_{1+1/2} \quad \text{(P244.47)}$$

$$B_{Q1} = 1 - A_{Q1} - C_{Q1} \quad \text{(P244.48)}$$

and the explicit increment to q_{W1} due to turbulent mixing in (P244.45) is given by

$$(\delta T_{W1})_{ex} = (g \delta t / \Delta p_1)(F^n_{qW}(2) - E^n_*) \quad \text{(P244.49)}$$

The increments to the total water contents for $2 \le k \le BL\_LEVELS$ are given by

$$A_{Qk} \delta q_{Wk-1} + B_{Qk} \delta q_{Qk} + C_{Qk} \delta q_{Wk+1} = (\delta q_{Wk})_{ex} + (\delta q_{Wk})_{nt} \quad \text{(P244.50)}$$

where

$$A_{Qk} = \gamma_k (g \delta t / \Delta p_k) RK_H(k) / \Delta z_{k-1/2} \quad \text{(P244.51)}$$

$$C_{Qk} = \gamma_{k+1} (g \delta t / \Delta p_k) RK_H(k+1) / \Delta z_{k+1/2} \quad \text{(P244.52)}$$

$$B_{Qk} = 1 - A_{Qk} - C_{Qk} \quad \text{(P244.53)}$$

and the explicit increment to q_{Wk} due to turbulent mixing is given by

$$(\delta q_{Wk})_{ex} = (g \delta t / \Delta p_k)(F^n_{qW}(k+1) - F^n_{qW}(k)) \quad \text{(P244.54)}$$

For $k = BL\_LEVELS$ the assumption that the flux at the top of layer BL_LEVELS is zero gives the following equation in place of (P244.50):

$$A_{Qk}\delta q_{Wk-1} + B_{Qk}\delta q_{Qk} = (\delta q_{Wk})_{ex} + (\delta q_{Wk})_{nt} \quad \text{(P244.55)}$$

where

$$A_{Qk} = \gamma_k (g\delta t/\Delta p_k) RK_H(k) / \Delta z_{k-1/2} \quad \text{(P244.56)}$$

$$B_{Qk} = 1 - A_{Qk} \quad \text{(P244.57)}$$

and the explicit increment to q_{Wk} due to turbulent mixing is given by

$$(\delta q_{Wk})_{ex} = -(g\delta t/\Delta p_k) F^n_{qW}(k) \quad \text{(P244.58)}$$

where $k = BL\_LEVELS$ in (P244.55)-(P244.58).

Scheme 2: Non-local mixing in unstable conditions

(iii).2 The equation for turbulent mixing of heat

Equations (P243.23) and (P243.24) still apply for Scheme 2 but, as outlined in Section P243(i), the total turbulent flux of T_L from layer $k - 1$ to layer for $2 \le k \le N_{rml}$ is given by

$$F_{TL}(k) = F^{(rm)}_{TL}(k) + F^{(lm)}_{TL}(k) \quad \text{(P244.201)}$$

where $F^{(rm)}_{TL}(k)$ is the non-local, "rapid mixing" (rm), flux of T_L and $F^{(lm)}_{TL}(k)$ is the "local mixing" (lm) flux. If $N_{rml} < 2$ all mixing is local. The finite difference form of (P243.301) is, for $2 \le k \le N_{rml}$,

$$F^{(rm)}_{TL}(k) = \frac{(p_{Nrml+1/2} - p_{k-1/2}) F_{TL}(1) + (p_{k-1/2} - p_*) F_{TL}(N_{rml}+1)}{(p_{Nrml+1/2} - p_*)}$$

(P244.202)

For $3 \le k \le N_{rml}$ (P244.202) can be rewritten as

$$F_{TL}^{(rm)}(k) = F_{TL}^{(rm)}(k-1) + \frac{\Delta p_{k-1}}{\Delta p_{rml}}\Big(F_{TL}(N_{rml}+1) - F_{TL}(1)\Big) \qquad \text{(P244.203)}$$

where Δp_{rml} is defined by (P243.203). (P244.203) is also valid for $k = 2$ if $F_{TL}^{(rm)}$ is interpreted as $F_{TL}(1) \equiv H_*/c_P$.

The finite difference form of (P243.2) is, again for $2 \le k \le N_{rml}$,

$$F_{TL}^{(lm)}(k) = -RK_H(k)\Big((T_{Lk} - T_{Lk-1})/\Delta z_{k-1/2} + g/c_P\Big) \qquad \text{(P244.204)}$$

where $RK_H(1)$ is given by (P243.142). The explicit local mixing flux is given by (P244.204) with timelevel n values of T_L . This is calculated in subcomponent P243.

So (P244.24) gives

$$\begin{aligned}\delta T_{L1} &= (g\delta t/\Delta p_1)\Big(F_{TL}^{(rm)}(2) + F_{TL}^{(lm)}(2) - F_{TL}(1)\Big) + (\delta T_{L1})_{nt} \\ &= (\delta T_{L1})_{lm} + (\delta T_L)_{rm} + (\delta T_{L1})_{nt}\end{aligned} \qquad \text{(P244.205)}$$

where

$$(\delta T_{L1})_{lm} = (g\delta t/\Delta p_1)\, F_{TL}^{(lm)}(2) \qquad \text{(P244.206)}$$

$$(\delta T_L)_{rm} = (g\delta t/\Delta p_{rml})\Big(F_{TL}(N_{rml}+1) - H_*/c_P\Big) \qquad \text{(P244.207)}$$

Similarly (P244.32) $2 \le k \le N_{rml}-1$ gives

$$\begin{aligned}\delta T_{Lk} &= (g\delta t/\Delta p_K)\,(F_{TL}^{(lm)}(k-1) - F_{TL}^{(lm)}(k) + F_{TL}^{(rm)}(k+1) - F_{TL}^{(rm)}(k)) + (\delta T_{Lk})_{nt} \\ &= (\delta T_{Lk})_{lm} + (\delta T_L)_{rm} + (\delta T_{Lk})_{nt}\end{aligned} \qquad \text{(P244.208)}$$

where

$(\delta T_L)_{rm}$ is given by (P244.207)

$$(\delta T_{Lk})_{lm} = (g\delta t/\Delta p_k)\left(F_{TL}^{(lm)}(k+1) - F_{TL}^{(lm)}(k)\right) \quad \text{(P244.209)}$$

For $k = N_{rml}$,

$$\begin{aligned}\delta T_{Lk} &= (g\delta t/\Delta p_k)\left(F_{TL}(k+1) - F_{TL}^{(lm)}(k) - F_{TL}^{(rm)}(k)\right) + (\delta T_{Lk})_{nt} \\ &= (\delta T_{Lk})_{lm} + (\delta T_L)_{rm} + (\delta T_{Lk})_{nt}\end{aligned}$$

(P244.210)

where

$(\delta T_L)_{rm}$ is again given by (P244.207)

$$(\delta T_{Lk})_{lm} = -(g\delta t/\Delta p_k)\, F_{TL}^{(lm)}(k) \quad \text{(P244.211)}$$

Now for $2 \le k \le N_{rml}-1$,

$$\begin{aligned}(\delta T_{Lk})_{lm+nt} &\equiv (\delta T_{Lk})_{lm} + (\delta T_{Lk})_{nt} \\ &= (g\delta t/\Delta p_k)\left(F_{TL}^{(lm)}(k+1) - F_{TL}^{(lm)}(k)\right) + (\delta T_{Lk})_{nt}\end{aligned}$$

(P244.212)

The implicit numerical scheme involves specifying the local mixing fluxes for $2 \le k \le N_{rml}$ as

$$\begin{aligned}F_{TL}^{(lm)}(k) &= F_{TL}^{(lm)n}(k) - \gamma_k R K_H(k)\,\big((\delta T_{Lk})_{lm} + (\delta T_L)_{rm} + (\delta T_{Lk})_{nt} \\ &\qquad - (\delta T_{Lk-1})_{lm} - (\delta T_L)_{rm} - (\delta T_{Lk-1})_{nt}\big)/\Delta z_{k-1/2} \\ &= F_{TL}^{(ln)n}(k) - \gamma_k R K_H(k)\left((\delta T_{Lk})_{lm+nt} - (\delta T_{Lk-1})_{lm+nt}\right)/\Delta z_{k-1/2}\end{aligned}$$

(P244.213) Note

that the rapid mixing increment to T_L cancels in the expression for the local mixing fluxes since it is uniform within the boundary layer. Substituting for $F_{TL}^{(lm)}$ from (P244.213) in (P244.212) gives

for $2 \le k \le N_{rml}-1$,

$$A_{Tk}(\delta T_{Lk+1})_{lm+nt} + B_{Tk}(\delta T_{Lk})_{lm+nt} + C_{Tk}(\delta T_{Lk-1})_{lm+nt} = (\delta T_{Lk})_{lm,ex} + (\delta T_{Lk})_{nt} \tag{P244.214}$$

where A_{Tk}, B_{Tk}, C_{Tk} are given by (P244.35-37) and

$$(\delta T_{Lk})_{lm.ex} = (g\delta t/\Delta p_k)\left(F_{TL}^{(lm)n}(k+1) - F_{TL}^{(lm)n}(k)\right) \tag{P244.215}$$

Similarly $(\delta T_{L1})_{lm+nt} \equiv (\delta T_{L1})_{lm} + (\delta T_{L1})_{nt}$ satisfies

$$A_{T1}(\delta T_{L2})_{lm+nt} + B_{T1}(\delta T_{L1})_{lm+nt} = (\delta T_{L1})_{lm.ex} + (\delta T_{L1})_{nt} \tag{P244.216}$$

where A_{T1} is given by (P244.28),

$$B_{T1} = 1 - A_{T1} \tag{P244.217}$$

$$(\delta T_{L1})_{lm,ex} = (g\delta t/\Delta p_1)F_{TL}^{(lm)n}(2) \tag{P244.218}$$

Also for $k = N_{rml}$, $(\delta T_{Lk})_{lm+nt} \equiv (\delta T_{Lk})_{lm} + (\delta T_{Lk})_{nt}$ satisfies

$$B_{Tk}(\delta T_{Lk})_{lm+nt} + C_{Tk}(\delta T_{Lk-1})_{lm+nt} = (\delta T_{Lk})_{lm.ex} + (\delta T_{Lk})_{nt} \tag{P244.219}$$

where C_{Tk} is given by (P244.36),

$$B_{Tk} = 1 - C_{Tk} \tag{P244.220}$$

$$(\delta T_{Lk})_{lm,ex} = 1(g\delta t/\Delta p_k) F_{TL}^{(lm)n}(k) \qquad \text{(P244.221)}$$

The problem of finding $(\delta T_{Lk})_{lm,ex}$ for $1 \preceq k \preceq N_{rml}$ $(N_{rml} \succeq 2)$ is therefore self-contained; solutions can be found without knowing $(\delta T_L)_{rm}$, δT_* or δT_{Lk} for $N_{rml}+1 \preceq k \preceq BL\_LEVELS$. The local mixing within the boundary layer diffusively smooths the profile; with the numerical scheme described above this is done conservatively.

Substituting in equation (P244.207) for H_* from (P244.11), using (P244.205) to express δT_{L1} as the sum of its local mixing, non-turbulent and rapid mixing parts, and for $F_{TL}(N_{rml}+1)$ from (P244.33), using (P244.210) to write $\delta T_{L,Nrml}$ as the sum of its three parts, gives

$$A_{Trml}\delta T_{L,Nrml+1} + B_{Trml}(\delta T_L)_{rm} + C_{Trml}\delta T_* = (g\delta t/\Delta p_{rml})(\hat{F}_{TL}(N_{rml}+1) - \hat{H}_*/c_P) \qquad \text{(P244.222) where}$$

$$A_{Trml} = \gamma_{Nrml+1}(g\delta t/\Delta p_{rml})RK_H(N_{rml}+1)/\Delta z_{Nrml+1/2} \qquad \text{(P244.223)}$$

$$C_{Trml} = \gamma_1(g\delta t/\Delta p_{rml})RK_H(1) \qquad \text{(P244.224)}$$

$$B_{Trml} = 1 - A_{Trml} - C_{Trml} \qquad \text{(P244.225)}$$

$$\hat{F}_{TL}(N_{rml}+1) = F_{TL}^n(N_{Rml}+1) + \gamma_{Nrml+1}(RK_H(N_{rml}+1)/\Delta z_{Nrml+1/2}\left(\delta T_{L,Nrml}\right)_{lm+nt} \qquad \text{(P244.226)}$$

$$\hat{H}_* = H_*^n - \gamma_1 c_P RK_H(1)\left(\delta T_{L1}\right)_{lm+nt} \qquad \text{(P244.227)}$$

Equation (P244.32) with $k = N_{rml}+1$ involves $F_{TL}(N_{rml}+1)$ and hence $\delta T_{L,Nrml}$. The latter is again written as the sum of its three parts to obtain, for $k = N_{rml}+1 < BL\_LEVELS$

$$A_{Tk}\delta T_{Lk+1} + B_{Tk}\delta T_{Lk} + C_{Tk}(\delta T_L)_{rm} = (\delta T_{Lk})_{ex} + (\delta T_{Lk})_{nt}$$

(P244.228)

where A_{Tk}, C_{Tk}, B_{Tk} are given by (P244.35-37) respectivel and

$$(\delta T_{Lk})_{ex} = (g\delta t/\Delta p_k)(F^n_{TL}(k+1) - \hat{F}_{TL}(k)) \quad \text{(P244.229)}$$

with $\hat{F}_{TL}(N_{rml}+1)$ given by (P244.226). For $k = N_{rml}+1 < BL\_LEVELS$,

$$B_{Tk}\delta T_{Lk} + C_{Tk}(\delta T_L)_{rm} = (\delta T_{Lk})_{ex} + (\delta T_{Lk})_{nt} \quad \text{(P244.230)}$$

where C_{Tk} and B_{Tk} are given by (P244.40,41) respectively and

$$(\delta T_{Lk})_{ex} = -(g\delta t/\Delta p_k)\hat{F}_{TL}(k) \quad \text{(P244.231)}$$

with $\hat{F}_{TL}(N_{rml})+1)$ again given by (P244.226).

The equations for $\hat{F}_{TL}(N_{rml}+1)$ for $N_{rml}+2 \le k \le BL\_LEVELS$ (only required when $N_{rml} \le BL\_LEVELS - 2$) are the same as those for Scheme 1, i.e. (P244.34-42), since only local mixing is done above the rapid mixing layer.

(iv).2 <u>The equation for turbulent mixing of moisture</u>

The finite difference form of equation (P243.46) still applies for Scheme 2 but, as outlined in Section P243(i), the total turbulent flux of q_W from layer $k - 1$ to layer k for $2 \le k \le N_{rml}$ is given by

$$F_{qW}(k) = F^{(rm)}_{qW}(k) + F^{(lm)}_{qW}(k) \quad \text{(P244.232)}$$

where $F^{(rm)}_{qW}(k)$ is the non-local, "rapid mixing" (rm), flux of q_W and $F^{(lm)}_{qW}(k)$ is the "local mixing" (lm) flux. If $N_{rml} < 2$ all mixing is local. The finite difference form of (P243.301) is,

for $2 \le k \le N_{rml}$,

$$F_{qW}^{(rm)}(k) = \frac{(p_{Nrml+1/2} - p_{k-1/2})F_{qW}(1) + (p_{k-1/2} - p_*)F_{qW}(N_{rml}+1)}{(p_{Nrml+1/2} - p_*)}$$

(P244.233)

For $3 \le k \le N_{rml}$ (P244.233) can be rewritten as

$$F_{qW}^{(rm)}(k) = F_{qW}^{(rm)}(k-1) + \frac{\Delta p_{k-1}}{\Delta p_{rml}}\left(F_{qW}(N_{rml}+1) - F_{qw}(1)\right) \quad \text{(P244.234)}$$

where Δp_{rml} is defined by (P243.203). (P244.234) is also valid for $k = 2$ if $F_{qW}^{(rm)}(1)$ is interpreted as $F_{qW}(1) \equiv E_*$.

The finite difference form of (P243.2) is, again for $2 \le k \le N_{rml}$,

$$F_{qW}^{(lm)}(k) = -RK_H(k)\left((q_{Wk} - q_{Wk-1})/\Delta z_{k-1/2} + g/c_P\right) \quad \text{(P244.235)}$$

where $RK_H(k)$ is given by (P243.142). The explicit local mixing flux is given by (P244.235) with timelevel n values of q_W . This is calculated in subcomponent P243.

The analogous equation to (P244.205) is

$$\begin{aligned}\delta q_{W1} &= (g\delta t/\Delta p_1)\left(F_{qW}^{(rm)}(2) + F_{qW}^{(lm)}(2) - E_*\right) + (\delta q_{W1})_{nt} \\ &= (\delta q_{W1})_{lm} + (\delta q_W)_{rm} + (\delta q_{W1})_{nt}\end{aligned}$$

(P244.236) where

$$(\delta q_{W1})_{lm} = (g\delta t/\Delta p_1)F_{qW}^{(lm)}(2) \quad \text{(P244.237)}$$

$$(\delta q_W)_{rm} = (g\delta t/\Delta p_{rml})\left(F_{qW}(N_{rml}+1) - E_*\right) \tag{P244.238}$$

Similarly for $2 \preceq k \preceq N_{rml} - 1$,

$$\delta q_{Wk} = (g\delta t/\Delta p_k)\left(F_{qW}^{(lm)}(k+1) - F_{qW}^{(lm)}(k) + F_{qW}^{(rm)}(k+1) - F_{qW}^{(rm)}(k)\right) + (\delta q_{Wk})_{nt}$$
$$= (\delta q_{Wk})_{lm} + (\delta q_W)_{rm} + (\delta q_{Wk})_{nt}$$

(P244.239)

where

$(\delta q_W)_{rm}$ is given by (P244.238)

$$(\delta q_{Wk})_{lm} = (g\delta t/\Delta p_k)\left(F_{Qw}^{(lm)}(k+1) - F_{qW}^{(lm)}(k)\right) \tag{P244.240}$$

For $k = N_{rml}$,

$$\delta q_{Wk} = (g\delta t/\Delta p_k)\left(F_{qW}^{(lm)} - F_{qW}^{(lm)}(k) - F_{qW}^{(rm)}(k)\right) + (\delta q_{Wk})_{nt}$$
$$= (\delta q_{Wk})_{lm} + (\delta q_W)_{rm} + (\delta q_{Wk})_{nt}$$

(P244.241)

where

$(\delta q_W)_{rm}$ is again given by (P244.238)

$$(\delta q_{Wk})_{lm} = -(g\delta t/\Delta p_k)\, F_{qW}^{(lm)}(k) \tag{P244.242}$$

Now for $2 \preceq k \preceq N_{rml}-1$,

$$(\delta q_{Wk})_{lm+nt} \equiv (\delta q_{Wk})_{nt}$$
$$= (g\delta t/\Delta p_k)\left(F_{qW}^{(lm)}(k+1) - F_{qW}^{(lm)}(k)\right) + (\delta q_{Wk})_{nt}$$

(P244.243)

The implicit numerical scheme involves specifying the local mixing fluxes for $2 \le k \le N_{rml}$ as

$$F_{qW}^{(lm)}(k) = F_{qW}^{(lm)n}(k) - \gamma_k R K_H(k)((\delta q_{Wk})_{lm} + (\delta q_{Wk})_{lm} + (\delta q_{Wk})_{nt} - (\delta q_{Wk-1})_{lm} - (\delta q_W)_{rm} - (\delta q_{Wk-1})_{nt} / \Delta z_{k-1/2}$$
$$F_{qW}^{(lm)n}(k) - \gamma_k R K_H(k)((\delta q_{Wk})_{lm+nt} - (\delta q_{Wk-1})_{lm+nt}) / \Delta z_{k-1/2}$$

(P244.244)

Note that the rapid mixing increment to q_W cancels in the expression for the local mixing fluxes since it is uniform within the boundary layer. Substituting for $F_{qW}^{(lm)}$ from (P244.244) in (P244.243) gives for $2 \le k \le N_{rml} - 1$,

$$A_{Qk}(\delta q_{Lk-1})_{lm+nt} + B_{Qk}(\delta q_{Wk})_{lm+nt} + C_{Qk}(\delta T_{Lk+1})_{lm+nt} = (\delta q_{Wk})_{lm,ex} + (\delta q_{Wk})_{nt} \quad \text{(P244.245)}$$

where A_{Qk}, B_{Qk}, C_{Qk} are given by (P244.51-53) and

$$(\delta q_{Wk})_{lm,ex} = (g \delta t / \Delta p_k) \left(F_{qw}^{(lm)n}(k+1) - F_{qW}^{(lm)n}(k) \right) \quad \text{(P244.246)}$$

Similarly $(\delta q_{W1})_{lm+nt} \equiv (\delta q_{W1})_{lm} + (\delta q_{W1})_{nt}$ satisfies

$$B_{Q1}(\delta w_{W1})_{lm+nt} + C_{Q1}(\delta q_{W2})_{lm+nt} = (\delta q_{W1})_{lm,ex} + (\delta q_{W1})_{nt} \quad \text{(P244.247)}$$

where C_{Q1} is given by (P244.47),

$$B_{Q1} = 1 - A_{Q1} \quad \text{(P244.248)}$$

$$(\delta q_{W1})_{lm,ex} = (g\delta t/\Delta p_1) F_{qW}^{(lm)n}(2) \quad \text{(P244.249)}$$

Also for $k = N_{rml}$, $(\delta q_{Wk})_{lm+nt} \equiv (\delta q_{Wk}) + (\delta q_{Wk})_{nt}$ satisfies

$$A_{Qk}(\delta q_{Wk-1})_{lm+nt} + B_{Qk}(\delta q_{Wk})_{lm+nt} = (\delta q_{Wk})_{lm,ex} + (\delta q_{Wk})_{nt} \quad \text{(P244.250)}$$

where A_{Qk} is given by (P244.51),

$$B_{Qk} = 1 - A_{Qk} \quad \text{(P244.251)}$$

$$(\delta q_{Wk})_{lm,ex} = -(g\delta t/\Delta p_k) F_{qW}^{(lm)n}(k) \quad \text{(P244.252)}$$

The problem of finding $(\delta q_{Wk})_{lm+nt}$ for $1 \preceq k \preceq N_{rml}$ $(N_{rml} \succeq 2)$ is therefore self-contained; solutions can be found without knowing $(\delta q_W)_{rm}$, δT_* or δq_{Wk} for $N_{rml}+1 \preceq k \preceq BL\_LEVELS$. The local mixing within the boundary layer diffusively smooths the profile.

Substituting in equation (P244.238) for E_* from (P244.12), using (P244.236) to express δq_{W1} as the sum of its local mixing, non-turbulent and rapid mixing parts, and for $F_{qW}(N_{rml}+1)$ from (P244.44), using (P244.241) to write $\delta q_{W,Nrml}$ as the sum of its three parts, gives

$$\alpha_*^n A_{Qrml}\delta T_* + B_{Qrml}(\delta q_W)_{rm} + C_{Qrml}\delta q_{W,Nrml+1} = (g\delta t/\Delta p_{rml})\left(\hat{F}_{qW}(N_{rml}+1) - \hat{E}_*\right) \quad \text{(P244.249)}$$

where

$$A_{Qrml} = \gamma_1 (g\delta t/\Delta p_{rml}) RK_E \quad \text{(P244.250)}$$

$$C_{Qrml} = \gamma_{Nrml+1}(g\delta t/\Delta p_{rml}) RK_H(N_{rml}+1)/\Delta z_{Nrml+1/2} \quad \text{(P244.251)}$$

$$B_{Qrml} = 1 - A_{Qrml} - C_{Qrml} \quad \text{(P244.252)}$$

$$\hat{F}_{qW}(N_{rml}+1) = F^n_{qW}(N_{rml}+1) + \gamma_{Nrml+1}(RK_H(N_{rml}+1)/\Delta z_{Nrml+1/2}\left(\delta q_{W,Nrml}\right)_{lm+nt} \quad \text{(P244.253)}$$

$$\hat{E}_* = E^n_* - \gamma_1 RK_E\left(\delta q_{W1}\right)_{lm+nt} \quad \text{(P244.254)}$$

The finite difference form of equation (P244.43) with $k = N_{rml}+1$ involves $F_{qW}(N_{rml}+1)$ and hence $\delta q_{W,Nrml}$. The latter is again written as the sum of its three parts to obtain, for $k = N_{rml}+1 < BL\_LEVELS$,

$$A_{Qk}(\delta q_W)_{rm} + B_{Qk}\delta q_{Wk} + C_{Qk}\delta q_{Lk+1} = (\delta q_{Wk})_{ex} + (\delta q_{Wk})_{nt} \quad \text{(P244.255)}$$

where A_{Qk}, C_{Qk}, B_{Qk} are given by (P244.51-53) respectively and

$$(\delta q_{Wk})_{ex} = (g\delta t/\Delta p_k)(F^n_{qW}(k+1) - \hat{F}_{qW}(k)) \quad \text{(P244.256)}$$

with $\hat{F}_{qW}(N_{rml}+1)$ given by (P244.253). For $k = N_{rml}+1 = BL\_LEVELS$,

$$A_{Qk}(\delta q_W)_{rm} + B_{Qk}\delta q_{Wk} = (\delta q_{Wk})_{ex} + (\delta q_{Wk})_{nt} \quad \text{(P244.257)}$$

where A_{Qk} and B_{Qk} are given by (P244.56,57) respectively and

$$(\delta q_{Wk})_{ex} = -(g\delta t/\Delta p_k)\hat{F}_{qW}(k) \quad \text{(P244.258)}$$

with $\hat{F}_{qW}(N_{rml}+1)$ again given by (P244.253).

The equations for δq_{Wk} for $N_{rml}+1 \le k \le BL\_LEVELS$ (only required when

$N_{rml} \le BL\_LEVELS - 2$) are the same as those for Scheme 1, i.e. (P244.50-58),

since only local mixing is done above the rapid mixing layer.

(ii).2 The surface temperature equation

Writing δT_{L1} and δq_{W1} as sums of local mixing, rapid mixing and non-turbulent parts

when $N_{rml} \ge 2$ and substituting in (P244.14) gives, for land points

$$A_{T*}(\delta T_L)_{rm} + B_{T*}\delta T_* + C_{T*}(\delta q_W)_{rm} = (\delta T_*)_{ex} \qquad \text{(P244.259)}$$

where A_{T*}, B_{T*} and C_{T*} are still given by (P244.16-18) and

$$(\delta T_*)_{ex} = \delta t A_{S1}(R_{N\downarrow *} - H_S - (\hat{H}_* + L\hat{E}_*)) \qquad \text{(P244.260)}$$

$\hat{H}_*$ and $\hat{E}_*$ are given by (P244.227) and (P244.254) respectively.

For sea points with sea-ice $(f_I > 0)$ the equation for δT_* becomes

$$f_I A_{T*}(\delta T_L)_{rm} + B_{T*}\delta T_* + f_I C_{T*}(\delta q_W)_{rm} = (\delta T_*)_{ex} \qquad \text{(P244.261)}$$

where A_{T*}, B_{T*} and C_{T*} are still given by (P244.20-22) and

$$(\delta T_*)_{ex} = \delta t A_I(R_{N\downarrow(I)*} - H_I - (\hat{H}_{*(I)} + (L_C+L_F)\hat{E}_{*(I)})) \qquad \text{(P244.262)}$$

$$\hat{H}_{*(I)} = H_{*(I)} - \gamma_1 c_P R K_H(1) f_I(\delta T_{L1})_{lm+nt} \qquad \text{(P244.263)}$$

$$\hat{E}_{*(I)} = E^n_{*(I)} - \gamma_1 R K_E f_I(\delta q_{W1})_{lm+nt} \qquad \text{(P244.264)}$$

Area weighted surface heat and moisture fluxes for the leads can also be defined consistently with (P244.263,264):

$$\hat{H}_{*(L)} = H^n_{*(L)} - \gamma_1 c_P R K_H(1)(1-f_I)(\delta T_{L1})_{lm+nt} \qquad \text{(P244.265)}$$

$$\hat{E}_{*(L)} = E^n_{*(L)} - \gamma_1 R K_E(1-f_I)(\delta q_{W1})_{lm+nt}$$

(P244.266) Schemes 1 and 2

(v) The equation for turbulent mixing of momentum in the boundary layer

Only local mixing of momentum is done but note that the Richardson numbers which are used in the calculation of the mixing coefficients $RK_M(k)$ are modified for $2 \le k \le N_{rml}$ according to the procedure described in subcomponent P243.

The prognostic equation for the vector horizontal wind is

$$\frac{\partial v}{\partial t} = -g\frac{\partial \underline{\tau}}{\partial p} + \left.\frac{\partial v}{\partial t}\right|_{nt} \qquad \text{(P244.59)}$$

The finite difference form of (P244.59) for atmospheric layer 1 is

$$\delta v_1 = -(g\delta t/\Delta p_1)(\underline{\tau}(2) - \underline{\tau_*} + (\delta v_1)_{nt} \qquad \text{(P244.60)}$$

where $\underline{\tau}(2)$ is the stress between layers 1 and 2 and $\underline{\tau_*} \equiv \underline{\tau}(1)$ is the surface stress. The implicit numerical integration scheme is defined by specifying these stresses as:

$$\begin{aligned}\underline{\tau_*} &= RK_M(1)(\gamma_1(v_1^{n+1} - v_0) + (1-\gamma_1)(v_1^0 - v_0) \\ &= \tau_*^n + \gamma_1 RK_M(1)\delta v_1\end{aligned} \qquad \text{(P244.61)}$$

and

$$\begin{aligned}\underline{\tau}(2) &= RK_M(2)(\gamma_2(v_2^{n+1} - v_1^{n+1})/\Delta z_{1+1/2} + (1-\gamma_2)(v_2^n - v_1^n)/\Delta z_{1+1/2} \\ &= \underline{\tau}^n(2) + \gamma_2 RK_M(2)(\delta v_2 - \delta v_1)/\Delta z_{1+1/2}\end{aligned}$$

(P244.62)

The timelevel n quantities $\underline{\tau_*^n}$, $RK_M(1)$, $\underline{\tau}^n(2)$ and $RK_M(2)$ are calculated in subcomponent P243 (see (P243.132, 133, 124, 143, 144, 141)).

Substituting for $\underline{\tau_*}$ and $\underline{\tau}(2)$ from (P244.61) and (P244.62) in (P244.60) gives

$$A_{M1}\delta v_2 + B_{M1}\delta v_1 = (\delta v_1)_{ex} + (\delta v_1)_{nt} \tag{P244.63}$$

where the dimensionless coefficients A_{M1} and B_{M1} are given by

$$A_{M1} = \gamma_2 (g\delta t/\Delta p_1) RK_M(2)/\Delta z_{1+1/2} \tag{P244.64}$$

$$C_{M1} + \gamma_1 (g\delta t/\Delta p_1) RK_M(1) \tag{P244.66}$$

$$B_{M1} = 1 - A_{M1} - C_{M1} \tag{P244.65}$$

and the "explicit" increment to v_1 is given by

$$(\delta v_1)_{ex} = -(g\delta t/\Delta p_1)(\underline{\tau}^n(2) - \underline{\tau}^n_*) \tag{P244.67}$$

Extending the same implicit treatment to the layers above layer 1, the finite difference form of the equation for v_k $(2 \le k \le BL\_LEVELS - 1)$ is

$$\delta v_k = -(g\delta t/\Delta p_k)(\underline{\tau}(k+1) - \underline{\tau}(k)) + (\delta v_k)_{nt} \tag{P244.68}$$

$\underline{\tau}(k)$ is the stress btween layers $k-1$ and layer k (for $2 \le k \le BL\_LEVELS - 1$). The implicit numerical integration scheme is defined by specifying this flux as:

$$\begin{aligned}\underline{\tau}(k) &= RK_M(k)(\gamma_k(v_k^{n+1} - v_{k-1}^{n+1})/\Delta z_{k-1/2} + (1-\gamma_k)(v_k^n - v_{k-1}^n)/\Delta z_{k-1/2} \\ &= \underline{\tau}^n(k) + \gamma_k RK_M(k)(\delta v_k - \delta v_{k-1})/\Delta z_{k-1/2}\end{aligned}$$

(P244.69)

The timelevel n quantities $RK_M(k)$ and $\underline{\tau}^n(k)$ are calculated in subcomponent P243 (see (P243.141, 139, 143, 144)). Substituting for the fluxes from (P244.69) in (P244.68) gives, for $2 \le k \le BL\_LEVELS - 1$,

$$A_{Mk}\delta v_{k+1} + B_{Mk}\delta v_k + C_{Mk}\delta v_{k-1} = (\delta v_k)_{ex} + (\delta v_k)_{nt} \qquad \text{(P244.70)}$$

where the dimensionless coefficients are given by

$$A_{Mk} = \gamma_{k+1}(g\delta t/\Delta p_k)RK_M(k+1)/\Delta z_{k+1/2} \qquad \text{(P244.71)}$$

$$C_{Mk} = \gamma_k(g\delta t/\Delta p_k)RK_M(k)/\Delta z_{k-1/2} \qquad \text{(P244.72)}$$

$$B_{Mk} = 1 - A_{Mk} - C_{Mk} \qquad \text{(P244.73)}$$

and the "explicit" increment to v_k is given by

$$(\delta v_k)_{ex} = -(g\delta t/\Delta p_k)(\underline{\tau}^n(k+1) - \underline{\tau}^n(k)) \qquad \text{(P244.74)}$$

For $k = BL\_LEVELS$ the only difference to the treatment for the layers below is in the assumption that the stress at the top of layer BL_LEVELS is zero.

This gives the following equation in place of (P244.70):

$$B_{Mk}\delta v_k + C_{Mk}\delta v_{k-1} = (\delta v_k)_{ex} + (\delta v_k)_{nt} \qquad \text{(P244.75)}$$

where

$$C_{Mk} = \gamma_k(g\delta t/\Delta p_k)RK_M(k)/\Delta z_{k-1/2} \qquad \text{(P244.76)}$$

$$B_{Mk} = 1 - C_{Mk} \qquad \text{(P244.77)}$$

and the "explicit" increment to v_k is given by

$$(\delta v_k)_{ex} = (g\delta t/\Delta p_k)\underline{\tau}^n(k) \qquad \text{(P244.78)}$$

where $k = BL\_LEVELS$ in (P244.75)-(P244.78).

(vi) Summary of the implicit turbulent mixing equations in matrix form and the method of solution

Scheme 1: Local mixing

The equations for the increments to T_{Lk} , T_* and q_{Wk} can be rewritten in matrix form as:

$$\begin{bmatrix}
B_{TN} & C_{TN} & & & & & & & \\
A_{TN-1} & B_{TN-1} & C_{TN-1} & & & & & 0 & \\
\ddots & \ddots & \ddots & & & & & & \\
 & A_{T1} & B_{T1} & C_{T1} & & & & & \\
 & & A_{T^*} & B_{T^*} & C_{T^*} & & & & \\
 & & & \alpha_*^n A_{Q1} & B_{Q1} & C_{Q1} & & & \\
 & 0 & & & \ddots & \ddots & \ddots & & \\
 & & & & & A_{QN-1} & B_{QN-1} & C_{QN-1} \\
 & & & & & & A_{QN} & B_{QN}
\end{bmatrix}
\begin{bmatrix}
\delta T_{LN} \\ \delta T_{LN-1} \\ : \\ \delta T_{L1} \\ \delta T_* \\ \delta q_{W1} \\ : \\ \delta q_{WN-1} \\ \delta_{WN}
\end{bmatrix}
=
\begin{bmatrix}
(\delta T_{LN})_{ex+nt} \\ (\delta T_{LN-1})_{ex+nt} \\ : \\ (\delta T_{L1})_{ex+nt} \\ (\delta T_*)_{ex} \\ (\delta q_{W1})_{ex+nt} \\ : \\ (\delta q_{WN-1})_{ex+nt} \\ (\delta q_{WN})_{ex+nt}
\end{bmatrix}$$

(P244.79)

where $N = BL\_LEVELS$. At sea-ice points A_{T^*}, C_{T^*} and α_*^n should be replaced

by $f_I A_{T^*}$, $f_I C_{T^*}$ and $\alpha^n_{*(I)}$ respectively in (P244.79).

Similarly the equations for the increments to v_k can be written as:

$$
\begin{bmatrix}
B_{MN} & C_{MN} & & \\
A_{MN-1} & B_{MN-1} & C_{MN-1} & 0 \\
\ddots & \ddots & \ddots & \\
 & \ddots & \ddots & \ddots \\
0 & A_{M2} & B_{M2} & C_{M2} \\
 & & A_{M1} & B_{M1}
\end{bmatrix}
\begin{bmatrix}
\delta v_N \\ \delta v_{N-1} \\ : \\ \delta v_2 \\ \delta v_1
\end{bmatrix}
=
\begin{bmatrix}
(\delta v_N)_{ex+nt} \\ (\delta v_{N-1})_{ex+nt} \\ : \\ (\delta v_2)_{ex+nt} \\ (\delta v_1)_{ex+nt}
\end{bmatrix}
$$

(P244.80)

The matrix coefficients and the "explicit" increments in (P244.79) and (P244.80) are defined in the preceding sections. The non-turbulent increments are not currently included on the right hand sides of (P244.79) and (P244.80). Instead they are added to their respective fields after they have been calculated in each of the other components of the atmospheric model. Janssen et al. (1992) have shown that an independent, or split, treatment of the model's processes can prevent proper physical balances becoming established. In particular they show that a numerical scheme that alculates boundary layer mixing and dynamical increments separately has an equilibrium that depends on timestep. Including the non-turbulent increments as forcing in the implicit equations for boundary layer mixing increments removes this deficiency. It is hoped to include the non-turbulent increments in the implicit scheme in a future version of the model. Note that in the surface temperature equation the full set of forcing increments are already included: $(\delta T_*)_{nt} = (\delta T_*)_{RN} + (\delta T_*)_{S(\ or\ I\ )F}$.

The required increments are calculated by inverting the two tridiagonal matrices by the method of Gaussian elimination. In an "upward sweep" the upper off-diagonal (or C) elements are eliminated by subtracting in turn a multiple of the row below and then each row is divided by the value of the resulting diagonal element. Then in a "downward sweep" the lower off-diagonal (or A) elements are eliminated bysubtracting in turn a multiple of the row above. All the operations are also applied to the right-hand side column matrices in (P244.79) and (P244.80). At the end of this procedure the right hand sides have become the required increments since the square matrices on the left hand sides have been been reduced to unit matrices.

In the code the calculation of the explicit increments and the A, B, C matrix elements is done in the same loop in which the upward sweep elimination is done. Since at the end of this loop the upper off-diagonal (or C) elements have been eliminated and the diagonal (or B) elements have been reduced to 1, only the modified lower off-diagonal (or A) elements and the right hand side increments need be temporarily stored for use in the second downward loop.

The algorithm for finding the δv_k is as follows:

Calculate:

$(\delta u_1)_{ex}$ and $(\delta v_1)_{ex}$ using (P244.67);

C_{M1} and A_{M1} using (P244.66) and (P244.64);

then set:

$$B_{M1} = 1 - A_{M1} - C_{M1} \quad \text{(P244.81)}$$

$$\delta u_1' = (\delta u_1)_{ex+nt} / B_{M1}$$

(P244.82)

$$\delta v_1' = (\delta v_1)_{ex+nt} / B_{M1}$$

(P244.83)

$$A_{M1}' = A_{M1} / B_{M1} \quad \text{(P244.84)}$$

(Comment: the division by B_{M1} normalises the diagonal element so B_{M1}' is now 1; only

$A_{M1}', \delta u_1', \delta v_1'$ are stored for later use.)

Next loop from $k = 2$ to $N - 1$ calculating:

$(\delta u_k)_{ex}$ and $(\delta v_k)_{ex}$ using (P244.74);

C_{MK} and A_{MK} using (P244.72) and (P244.71);

and setting:

$$B_{Mk}' = 1 - A_{Mk} - C_{Mk}(1 + A_{Mk-1}') \quad \text{(P244.85)}$$

$$\delta u_k' = \left((\delta u_k)_{ex+nt} - C_{Mk}\delta u_{k-1}' \right)/B_{Mk}' \quad \text{(P244.86)}$$

$$\delta v_k' = \left((\delta v_k)_{ex+nt} - C_{Mk}\delta v_{k-1}' \right)/B_{Mk}' \quad \text{(P244.87)}$$

$$A_{Mk}' = A_{Mk} / B_{Mk}' \qquad \text{(P244.88)}$$

(Comment: C_{MK} times primed row $k - 1$ has been subtracted from row k and then the diagonal element normalised to obtain the primed row k quantities in (P244.85) to (P244.88); C_{MK}' is now zero and the diagonal element is 1; $A_{Mk}', \delta u_k', \delta v_k'$ are stored for later use.)

For $k = N$ calculate:

$(\delta u_N)_{ex}$ and $(\delta v_N)_{ex}$ using (P244.78);

C_{MN} using (P244.76);

and set:

$$B_{MN}' = 1 - A_{MN} - C_{MN}(1 + A_{MN-1}') \qquad \text{(P244.89)}$$

$$\delta u_N = \left((\delta u_N)_{ex+nt} - C_{MN}\delta u_{N-1}' \right)/B_{MN}' \qquad \text{(P244.90)}$$

$$\delta v_N = \left((\delta v_N)_{ex+nt} - C_{MN}\delta v_{N-1}' \right)/B_{MN}' \qquad \text{(P244.91)}$$

(Comment: C_{MN} times primed row $N - 1$ has been subtracted from row N and then the diagonal element normalised to obtain the primed row N quantities in (P244.89) to (P244.91); C_{MN}' is now zero and the diagonal element is 1; note that since the off-diagonal element for row N is now zero and the diagonal element 1, the final implicit increments have now been found and are given by (P244.90) and (P244.91).)

Finally looping from $k = N - 1$ to 1 set:

$$\delta u_k = \delta u_k' - A_{MK}'\delta u_{k+1}$$

(P244.92)

$$\delta v_k = \delta v_k' - A_{MK}' \delta v_{k+1}$$

(P244.93)

(Comment: A_{MK}' times row $k+1$ has been subtracted to eliminate the A' matrix elements and obtain the final implicit increments.)

The updated wind components for $k = 1 \; to \; N$ are calculated using:

$$u_k^{n+1} = u_k^n + \delta u_k$$

(P244.94)

$$v_k^{n+1} = v_k^n + \delta v_k$$

(P244.95)

The wind components at 10 m above the surface can then be diagnosed, if required, using

$$u(10m) = u_0 + \overline{C_{DR}}(10m)(u_1^{n+1} - u_o) \quad \text{(P244.96)}$$

$$v(10m) = v_0 + \overline{C_{DR}}(10m)(v_1^{n+1} - v_o) \quad \text{(P244.97)}$$

which follow from (P243.81). The stress components at the surface and at the layer interfaces above the surface are calculated using (P244.61) and (P244.69).

The implicit increments for $T_{LK}, \; T_* \; and \; q_{Wk}$ are found by exactly the same double sweep elimination procedure applied to (P244.79). The algorithm is as follows:

Calculate:

$(\delta q_{WN})_{ex}$ using (P244.58) and A_{QN} using (P244.56);

and set:

$$B_{QN} = 1 - A_{QN} \quad \text{(P244.98)}$$

$$\delta q_{WN}' = (\delta q_{WN})_{ex+nt} / B_{QN} \quad \text{(P244.99)}$$

$$A_{QN}' = A_{QN} / B_{QN} \quad \text{(P244.100)}$$

(Comment: B_{QN}' is now 1; only A_{QN}' and $\delta q_{WN}'$ have to be stored for later use.)

Next loop from $k = N - 1$ down to 2 calculating:

$(\delta q_{Wk})_{ex}$ using (P244.54), A_{Qk} using (P244.51) and C_{Qk} using (P244.52);

and setting:

$$B_{QK}' = 1 - A_{QK} - C_{QK}(1 + A_{QK+1}') \qquad \text{(P244.101)}$$

$$\delta q_{Wk}' = \left((\delta q_{Wk})_{ex+nt} - C_{Qk}\delta Q_{Wk+1}'\right)/B_{Qk}' \qquad \text{(P244.102)}$$

$$A_{Qk}' = A_{Qk}/B_{QNk}' \qquad \text{(P244.103)}$$

(Comment: C_{Qk} times primed q-row $K + 1$ has been subtracted from q-row k and then the diagonal element normalised to obtain the primed row k quantities in (P244.101) to (P244.103); C_{Qk}' is now zero and the diagonal element is 1; only A_{Qk}' and $\delta q_{Wk}'$ have to be stored for later use.)

Now calculate:

$(\delta q_{W1})_{ex}$ using (P244.49);

A_{Q1} using (P244.46) and C_{Q1} using (P244.47);

and set:

$$B_{Q1}' = 1 - A_{Q1} - C_{Q1}(1 + A_{Q2}') \qquad \text{(P244.104)}$$

$$\delta q_{W1}' = \left((\delta q_{W1})_{ex+nt} - C_{Q1}\delta Q_{W2}'\right)/B_{Qk}' \qquad \text{(P244.105)}$$

$$A_{Q1}' = A_{Q1}/B_{Q1}'$$

(P244.106) (Comment: C_{Q1} times primed q-row 2 has been subtracted from q-row 1 and then the diagonal element normalised; C_{Q1}' is now zero and the diagonal element is 1;

only A_{Q1}' and $\delta q_{W1}'$ have to be stored for later use.)

For the middle (or T_*) row of (P244.79) calculate:

$(\delta T_*)_{ex}$ using (P244.15) for land points or (P244.19) for sea points with sea-ice,

(not required or calculated for sea points with no sea-ice);

α_*^n using (P244.13a) when $\left|(\delta T_*)_{ex}\right| \le 2\ K$ and (P244.13b) otherwise (at sea-ice

points $\alpha_{*(I)}^n$ using (P244.13a) when $\left|(\delta T_*)_{ex}/f_I\right| \le 2\ K$ and (P244.13b)

otherwise;

A_{T^*} using (P244.16) for land or (P244.20) for sea-ice, (not required or calculated for

sea points with no sea-ice);

C_{T^*} using (P244.17) for land or (P244.21) for sea-ice, (not required or calculated for

sea points with no sea-ice);

and set:

for land points:

$$B_{T^*}' = 1 - A_{T^*} - C_{T*}\alpha_*^n(1 + A_{Q1}') \qquad \text{(P244.107)}$$

$$\delta_{T^*}' = \left((\delta T_*)_{ex} - C_{T^*}\delta q_{W1}' \right) / B_{T^*}' \qquad \text{(P244.108)}$$

$$A_{T^*}' = A_{T^*}/B_{T^*}' \qquad \text{(P244.109)}$$

for sea-ice points:

$$B_{T^*}' = 1 - A_{T^*} - C_{T*}\alpha_{*(I)}^n(1 + fA_{Q1}')_{(I)} \qquad \text{(P2440.14)}$$

$$\delta T_*' = \left((\delta T_*)_{ex} - f_I C_{T^*}\delta q_{W1}'\right)/B_{T^*}' \qquad \text{(P2440.15)}$$

$$A_{T^*}' = f_I A_{T^*}/B_{T^*}' \qquad \text{(P2440.16)}$$

for sea points with no sea-ice:

$$\delta T_*' = 0$$
$$A_{T^*}' = 0$$

(Comment: C_{T^*} ($f_I C_{T^*}$ for sea-ice points) times primed q-row 1 has been subtracted from the T_* row; the diagonal element has then been normalised; C_{T^*}' is now zero; only A_{T^*}' and $\delta T_*'$ have to be stored for later use. A_{T^*}' in (P2440.16) has been multiplied by f_I here rather than in the "downward sweep" (see (P244.119) below) to save on logical branches.)

For the T-row 1 calculate:

$(\delta T_{L1})_{ex}$ using (P244.31);

A_{T1} using (P244.28) and C_{T1} using (P244.29);

then set:

$$B_{T1}' = 1 - A_{T1} - C_{T1}(1 + A_{T^*}') \qquad \text{(P244.110)}$$

$$\delta T_{L1}' = \left((\delta T_{L1})_{ex+nt} - C_{T1}\delta T_*'\right)/B_{T1}'$$

(P244.111) $A_{T1}' = A_{T1}/B_{T1}'$

(P244.112) (Comment: C_{T1} times the primed T_* row has been subtracted from T-row 1 and then the diagonal element normalised; C_{T1}' is now zero; only A_{T1}' and δT_{L1} have to be stored for later use.)

Then loop from $k = 2 \ to \ N-1$ calculating:

$(\delta T_{Lk})_{ex}$ using (P244.38);

A_{Tk} using (P244.35) and C_{Tk} using (P244.36);

and setting:

$$B_{Tk}' = 1 - A_{Tk} - C_{Tk}(1 + A_{Tk-1}') \quad \text{(P244.113)}$$

$$\delta T_{Lk}' = \left((\delta T_{Lk})_{ex+nt} - C_{Tk}\delta_{Lk-1}'\right)/B_{Tk}' \quad \text{(P244.114)}$$

$$A_{Tk}' = A_{Tk}/B_{Tk}'$$

(P244.115)

(Comment: C_{Tk} times primed T-row k-1 has been subtracted from T-row k-1 and then the diagonal element normalised; C_{Tk}' is now zero; only A_{Tk}' and $\delta T_{Lk}'$ have to be stored for later use.)

To complete the "upward sweep" calculate:

$(\delta T_{LN})_{ex}$ using (P244.42); and C_{TN} using (P244.40);

and set:

$$B_{TN}' = 1 - C_{TN}(1 + A_{TN-1}') \quad \text{(P244.116)}$$

$$\delta T_{LN}' = \left((\delta T_{LN})_{ex+nt} - C_{TN}\delta_{LN-1}'\right)/B_{TN}' \quad \text{(P244.117)}$$

(Comment: C_{TN} times primed T-row N-1 has been subtracted from T-row N and then the diagonal element normalised; since C_{TN}' is now zero and the diagonal element is 1 the final implicit increment for T_L has been found and is given by (P244.117).)

Looping from $k = N-1 \; to \; 1$ set:

$$\delta T_{Lk} = \delta T_{Lk}' - A_{Tk}'\delta T_{Lk+1} \tag{P244.118}$$

then set:

$$\delta T_* = \delta T_*' - A_*'\delta T_{L1} \tag{P244.119}$$

$$\delta q_{W1} = \delta q_{W1}' - A_{Q1}'\alpha_*^n\delta T_* \tag{P244.120}$$

and finally looping from $k = 2 \; to \; N$ set:

$$\delta q_{Wk} = \delta q_{Wk}' - A_{Qk}'\delta q_{Wk-1} \tag{P244.121}$$

The fluxes of sensible heat at the surface and at the layer interfaces above the surface are then calculated using (P244.11) and (P244.33); the area-weighted sea-ice and leads heat fluxes are calculated using (P2440.1) and (P2440.3).

The surface moisture fluxes are calculated using:

$$E_A = E_A^N - \gamma_1 RK_{EA}\Delta q \tag{P244.122}$$

$$E_S = E_S^N - \gamma_1 RK_{ES}\Delta q \tag{P244.123}$$

$$E_{SL} = E_{SL}^N - \gamma_1 RK_{ESL}\Delta q$$

(P244.124) $E_* = E_{*A} + E_{*S}$

(P244.125)

where

$$\Delta q = \delta q_{W1} = \alpha_*^n\delta T_* \tag{P244.126}$$

and the time level n values are those given by (P243.136 to 138); the area weighted sea-ice and leads moisture fluxes are calculated using (P2440.2) and (P2440.4). The moisture fluxes at the layer interfaces above the surface are calculated using (P244.44).

T_{Lk} and q_{Wk} are updated for $k = 1 \; to \; BL\_LEVELS$ using the increments

just calculated:

$$T_{Lk}^{n+1} = T_{Lk}^n + \delta T_{Lk} \tag{P244.127}$$

$$q_{Wk}^{n+1} = q_{Wk}^{n} + \delta q_{Wk} \qquad \text{(P244.128)}$$

T_* is also updated:

$$T_*^{n+1} = T_*^{n} + \delta T_* \qquad \text{(P244.129)}$$

Further increments to T_* and q_{W1} are calculated in component P245 (Surface evaporation and sublimation) if one or more of the surface water stores (snow amount, canopy water, soil moisture) has insufficient water to maintain the surface moisture flux calculated here during the whole timestep.

<u>Scheme 2: Non-local mixing in unstable conditions</u>

The equations for the increments to T_{Lk} and q_{Wk} due to local mixing within the boundary layer, i.e. for $1 \le k \le N_{rml}$ when $N_{rml} \ge 2$, can be summarised in matrix form by

$$\begin{bmatrix} B_{Nr} & C_{Nr} & & 0 \\ A_{Nr-1} & B_{Nr-1} & C_{Nr-1} & \\ \ddots & \ddots & \ddots & \\ & A_2 & B_2 & C_2 \\ 0 & & A_1 & B_1 \end{bmatrix} \begin{bmatrix} (\delta X_{Nr})_{lm+nt} \\ (\delta X_{Nr-1})_{lm+nt} \\ \vdots \\ (\delta X_2)_{lm+nt} \\ (\delta X_1)_{lm+nt} \end{bmatrix} = \begin{bmatrix} (\delta X_{Nr})_{lm(ex)+nt} \\ (\delta X_{Nr-1})_{lm(ex)+nt} \\ \vdots \\ (\delta X_2)_{lm(ex)+nt} \\ (\delta X_1)_{lm(ex)+nt} \end{bmatrix}$$

(P244.267) where N_{rml} has been abreviated to N_r and

$$A_k = A_{Tk} = C_{Qk} \quad 1 \le k \le N_{rml} - 1$$

(P244.268) $C_k = C_{Tk} = A_{Qk} \quad 1 \le k \le N_{rml}$

(P244.269) $X_k = (T_{Lk}, q_{Wk})$

The tridiagonal matrix is inverted by Gaussian elimination just as for (P244.80). The loops have to range from 1 to BL_LEVELS - 1 since N_{rml} can have values up to BL_LEVELS - 1. Only if $N_{rml} \ge 2$ are $(\delta X_1)_{1m(ex)}$, A_1 and B_1 calculated. In the upward loop from $k = 2$ to BL_LEVELS - 1, if $k < N_{rml}$ then A_k, B_k, C_k and the explicit local mixing fluxes are calculated; if $k = N_{rml}$ then B_k, C_k and the explicit local mixing fluxes are calculated; if $k > N_{rml}$ no calculations are done.

In the downward loop, after the local mixing increments have been calculated, and if $k = N_{rml} > 2$, the modified top-of-boundary-layer fluxes $\hat{F}_{TL}(N_{rml}+1)$ and $\hat{F}_{qW}(N_{rml}+1)$ are calculated using (P244.226) and (P244.253) repectively. $k = 1$ and $N_{rml} > 2$ the following modified surface fluxes are calculated:

for land points:

$$\hat{E}_A = E_A^n - \gamma_1 R K_{EA} (\delta q_{W1})_{1m+nt}$$

(P244.270) $\hat{E}_S = E_S^n - \gamma_1 R K_{ES} (\delta q_{W1})_{1m+nt}$

(P244.271) $\hat{E}_{SL} = E_{SL}^n - \gamma_1 R K_{ESL} (\delta q_{W1})_{1m+nt}$

(P244.272) $\hat{E}_* = \hat{E}_A + \hat{E}_S$

(P244.273) $\hat{H}_*$ is given by (P244.227)

for sea points with sea-ice:

$$\hat{E}_{*(I)} = E^{n}_{*(I)} - \gamma_1 RK_E f_I (\delta q_{W1})_{lm+nt} \quad \text{(P244.274)}$$

$$\hat{H}_{*(I)} = H^{n}_{*(I)} - \gamma_1 c_P RK_H f_I (\delta q_{L1})_{lm+nt}$$

(P244.275) $$\hat{E}_{*(L)} = E^{n}_{*(L)} - \gamma_1 RK_E (1-f_I)(\delta q_{W1})_{lm+nt}$$

(P244.276) $$\hat{H}_{*(L)} = H^{n}_{*(L)} - \gamma_1 c_P \, RK_H(1)(1-f_I)(\delta q_{L1})_{lm+nt}$$

(P244.277) $$\hat{E}_{*} = \hat{E}_{*(I)} + \hat{E}_{*(L)}$$

(P244.278) $$\hat{H}_{*} = \hat{H}_{*(I)} + \hat{H}_{*(L)}$$

(P244.279) for sea points without sea-ice $\hat{H}_*$ and $\hat{E}_*$ are given by (P244.227) and (P244.254) respectively.

The modified fluxes are then used in the equations for the rapid mixing increments which are summarised by the matrix equation

$$\begin{bmatrix} B_{TN} & C_{TN} & & & & & & & \\ \ddots & \ddots & & & & 0 & & & \\ A_{TN_r+1} & B_{TN_r+1} & C_{TN_r+1} & & & & & & \\ & A_{Trml} & B_{Trml} & C_{Trml} & & & & & \\ & & A_{T*} & B_{T*} & C_{T*} & & & & \\ & & & \alpha^n_* A_{Qrml} & B_{Qrml} & C_{Qrml} & & & \\ & 0 & & & A_{QN_r+1} & B_{QN_r+1} & C_{QN_r+1} & & \\ & & & & & \ddots & \ddots & & \\ & & & & & & A_{QN} & B_{QN} & \end{bmatrix} \begin{bmatrix} (\delta T_{LN}) \\ \vdots \\ (\delta T_{LN_r+1}) \\ (\delta T_L)_{rm} \\ \delta T_* \\ (\delta q_W)_{rm} \\ \delta qWN_r+1 \\ \vdots \\ \delta Q_{WN} \end{bmatrix} = \begin{bmatrix} (\delta T_{LN})_{ex+nt} \\ \vdots \\ (\delta T_{LN_r+1})_{ex+nt} \\ (\delta T_L)_{rm,nt} \\ (\delta T_{*_1})_{ex} \\ (\delta q_W)_{rm,ex} \\ (\delta q_{WN_r+1})_{ex+nt} \\ \vdots \\ (\delta q_{WN})_{ex+nt} \end{bmatrix}$$

(P244.280)

N_{rml} has been abreviated to N_r and BL_LEVELS to N. At sea-ice

points A_{T*}, C_{T*} and α_*^n should be replaced by $f_I A_{T*}$, $f_I C_{T*}$ and $\alpha_{*(I)}^n$ respectively.

(P244.280) asssumes $1 < N_{rml} < BL\_LEVELS - 1$ but this need not be so. The

equations for $N_{rml} < k \leq BL\_LEVELS - 1$ do not exist

if $N_{rml} = BL\_LEVELS - 1$ and the equations for the rapid mixing increments do not exist

if $N_{rml} \leq 1$. These cases are accommodated by logical branches in both the upward and

downward sweep in the Gaussian elimination procedure.
As in Scheme 1 the non-turbulent increments are not passed into the boundary layer mixing subroutine in current versions of the model; they are added to their respective fields as they are calculated.

After the increments have been calculated the "implicit" fluxes which produced them are calculated.

For $N_{rml} + 2 \leq k \leq BL\_LEVELS$ when $N_{rml} \geq 2$ (if such k exist), and for

$2 \leq k \leq BL\_LEVELS$ when $N_{rml} < 2$,

$F_{TL}(k)$ is calculated using (P244.33)

$F_{qW}(k)$ is calculated using (P244.44)

For $k = N_{rml} + 1$ when $N_{rml} \geq 2$,

$$F_{TL}(k) = F_{TL}^n(k) - \gamma_k RK_H(k)\left(\delta T_{Lk} - (\delta T_L)_{rm} - (\delta T_{Lk-1})_{lm+nt}\right)/\Delta z_{k-1/2}$$
$$= \hat{F}_{TL}(k) - \gamma_k RK_H(k)\left(\delta T_{Lk} - (\delta T_L)_{rm}\right)/\Delta z_{k-1/2}$$

(P244.281)

$$F_{qW}(k) = F^{n}_{qW}(k) - \gamma_k RK_H(k)\left(\delta q_{Wk} - (\delta T_W)_{rm} - (\delta q_{Wk-1})_{lm+nt}\right)/\Delta z_{k-1/2}$$
$$= \hat{F}_{qW}(k) - \gamma_k RK_H(k)\left(\delta q_{Wk} - (\delta q_W)_{rm}\right)/\Delta z_{k-1/2}$$

(P244.282)

For $2 \le k \le N_{rml}$,

$F^{(lm)}_{TL}(k)$ is calculated using (P244.213)

$F^{(lm)}_{qW}(k)$ is calculated using (P244.244)

The surface fluxes are:

for land points, if $N_{rml} \ge 2$,

$$H_* = H^n_* - \gamma_1 c_P RK_H(1)\left((\delta T_L)_{rm} + (\delta T_{L1})_{lm+nt} - \delta T_*\right)$$
$$= \hat{H}_* - \gamma_1 c_P RK_H(1)\left((\delta T_L)_{rm} - \delta T_*\right) \quad \text{(P244.283)}$$

$$E_A = \hat{E}_A - \gamma_1 RK_E\left((\delta q_W)_{rm} - \alpha^n_* \delta T_*\right) \quad \text{(P244.284)}$$

$$E_S = \hat{E}_S - \gamma_1 RK_{ES}\left((\delta q_W)_{rm} - \alpha^n_* \delta T_*\right) \quad \text{(P244.285)}$$

$$E_{SL} = \hat{E}_{SL} - \gamma_1 RK_{ESL}\left((\delta q_W)_{rm} - \alpha^n_* \delta T_*\right) \quad \text{(P244.286)}$$

$$E_* = E_A + E_S \quad \text{(P244.287)}$$

if $N_{rml} < 2$ these fluxes are calculated using (P244.11) and (P244.122-126);

for sea points with sea-ice if $N_{rml} \ge 2$,

$$H_{*(L)} = \hat{H}_{*(L)} - \gamma_1 c_P RK_H(1)(1-f_I)(\delta T_L)_{rm} \quad \text{(P244.288)}$$

$$H_{*(I)} = \hat{H}_{*(I)} - \gamma_1 c_P RK_H(1)(f_I(\delta T_L)_{rm} - \delta T_*) \quad \text{(P244.289)}$$

$$E_{*(L)} = \hat{E}_{*(L)} - \gamma_1 \, RK_E(l)\,(1-f_I)\,(\delta q_W)_{rm}$$

(P244.290) $$E_{*(I)} = \hat{E}_{*(I)} - \gamma_1 \, RK_E \left(f_I (\delta q_W)_{rm} - \alpha^n_{*(I)} \; \delta T_* \right)$$

(P244.291) if $N_{rml} < 2$ these fluxes are calculated using (P2440.1-4);

for sea points without sea-ice if $N_{rml} \geq 2$,

$$H_* = \hat{H}_* - \gamma_1 c_P \, RK_H(l)\,(\delta T_L)_{rm} \qquad \text{(P244.292)}$$

$$E_* = \hat{E}_* - \gamma_1 \, RK_E (\delta q_W)_{rm} \qquad \text{(P244.293)}$$

if $N_{rml} < 2$ these fluxes are calculated using (P244.11) and (P244.12).

The total heat flux between layer $k-1$ and k for $2 < k \leq N_{rml}$ is

$$\begin{aligned} F_{TL}(k) &= F_{TL}^{(lm)}(k) + F_{TL}^{(rm)}(k) \\ &= F_{TL}^{(lm)}(k) + F_{TL}^{(rm)}(k-1) + \left(\Delta p_{k-1}/\Delta p_{rml}\right)\left(F_{TL}\left(N_{rml}+1\right) - H_*/c_P\right) \\ &= F_{TL}^{(lm)}(k) + F_{TL}^{(rm)}(k-1) + (\delta T_L)_{rm} / (g \delta t / \Delta p_{k-1}) \\ &= F_{TL}^{(lm)}(k) + F_{TL}(k-1) - F_{TL}^{(lm)}(k-1) + (\delta T_L)_{rm} / (g \delta t / \Delta p_{k-1}) \end{aligned}$$

(P244.294)

and for $k = 2$ (with $N_{rml} \geq 2$) is

$$F_{TL}(2) = F_{TL}^{(lm)}(2) + H_*/c_P + (\delta T_L)_{rm} / (g \delta t / \Delta p_1) \qquad \text{(P244.295)}$$

where use has been made of (P244.203). Similarly the moisture flux between layer $k-1$ and k for $2 < k \leq N_{rml}$ is

$$F_{qW}(k) = F_{qW}^{(lm)}(k) + F_{qW}(k-1) - F_{qw}^{(lm)}(k-1) + (\delta q_W)_{rm}/(g\delta t/\Delta p_{k-1})$$

(P244.296)

and for $k = 2$ (with $N_{rml} \geq 2$) is

$$F_{qW}(2) = F_{qW}^{(lm)}(2) + E_* + (\delta q_W)_{rm}/(g\delta t/\Delta p_1)$$

(P244.297) where use has been made of (P244.234).

(vii) Discussion on the value of the forward timelevel weighting factor

It is usual for the forward timelevel weighting factor, γ_k , to be in the range 0 to 1 with $\gamma_k = 0$ giving an explicit scheme and $\gamma_k = 1$ giving an implicit scheme with full weighting for the forward (or $n + 1$) timelevel. In the current version of the model the forward timelevel is "overweighted" for fluxes near the surface with γ_k set to 2 for $k = 1$ and 2, to 1.5 for $k = 3$ and to 1 for $k > 3$. $\gamma = 2$ equivalent to using scheme i (double timestep, explicit coefficient, implicit prognostic variable, followed by time average) of Kalnay and Kanamitsu (1988). Kalnay and Kanamitsu (1988) show that the overweighting is highly damping of large amplitude oscillations which can be produced because of the use of turbulent diffusion coefficients RK which depend on the timelevel n prognostic variables. However, Giraud and Delarge (1990) show that using a large constant value of γ can give an underestimate of the diffusion even for moderate values of δt . They show how to derive a (spatially and temporally) local value for γ which simultaneously achieves absolute stability and optimum accuracy. The Giraud and Delarge scheme has not been adopted because of the extra computational cost.

References

Giraud,C. and Delarge,Y., 1990: Stable schemes for nonlinear diffusion in atmospheric circulation models. Mon. Wea. Rev., **118**, 737-745.

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P245 Adjustment of the surface evaporation and sublimation

(subroutine SF_EVAP)

(i) Introduction

The surface evaporation and sublimation calculated in subcomponent P244 may imply that one or more of the land surface water stores (lying snow amount, canopy water and soil water) becomes exhausted during a timestep. For land points the purpose of this component is to detect when this would occur and adjust the moisture fluxes so that no more than the available snow or water enters the atmosphere as vapour. The latent heat flux, the surface temperature and the temperature and moisture in the boundary layer need to be adjusted correspondingly.

At sea points with sea-ice fraction greater than zero the ice surface temperature is reduced to the melting point of ice, T_M , if it has become greater. The latent heat flux associated with this implied melting is calculated.

(ii) The adjustment of land surface evaporation and sublimation

Several cases are considered:

a) Snow-free land with non-negative surface moisture flux.

It is assumed that all the water in the canopy and soil is liquid (so permafrost is not considered). This implies that all positive moisture flux from snow-free land is evaporation rather than sublimation even if the surface temperature, T_* , is less than or equal to T_M . The sublimation is therefore set to zero i.e.

$$E_I = 0 \qquad \text{(P245.1)}$$

The canopy evaporation, E_C , is initially set to E_A (given by (P244.122)). A test in then done to see if the canopy can sustain this evaporation through the whole timestep. If it cannot, i.e. if $E_C \delta T > C$, then the canopy evaporation and the soil evaporation E_S (given by (P244.123)) are adjusted. Clearly in this case evaporation from the canopy can only proceed at a rate E_C for a time $\delta t = C / E_C$ so the adjusted soil evaporation, E_S' , is given by

$$E_S' \delta t = E_S \delta t + E_{SL}(\delta t - \hat{\delta} t) \qquad \text{(P245.2)}$$

where the local soil evaporation E_{SL} is given by (P244.124). Substituting for δ in (P245.2) gives

$$E_S' = E_{SL} + \left(c \ / \ (E_C \ \delta t)\right)(E_S - E_{SL})$$
$$= \left(1 - f_A c \ / \ E_C \delta t)\right) E_{SL} \qquad \text{(P245.3)}$$

The adjusted canopy evaporation is given by

$$E_C' = c \ / \ \delta t \qquad \text{(P245.4)}$$

(so $E_C' \delta t = c = E_C \delta t$). If $E_C \delta t \le c$ no adjustment is necessary

so $E_S' = E_S$ and $E_C' = E_C$

A further test is done to see if the soil water store can sustain the soil evaporation through the whole timestep. If the soil moisture content, m , is less than or equal to zero there can clearly be no soil evaporation and if $E_S' \delta t > m$ the soil evaporation must be limited to $m \ / \ \delta t$.

So the algorithm for calculating E_S'' , the adjusted gridbox mean soil evaporation is

(P245.5)

$$E_S'' = \begin{cases} 0 & if \ m \le 0 \\ m/\delta t & if \ E_S' \delta t > m \\ E_S' & if \ E_S' \delta t \le m \end{cases} \qquad \text{(P245.6)}$$

Finally the total evaporation of surface and subsurface liquid water is set as

$$E_W = E_C' + E_S'' \qquad \text{(P245.7)}$$

b) **Snowfree land with negative surface moisture flux and surface temperature above freeezing.**

In this case there is condensation onto the canopy and so

$$E_C = E_* \quad (< 0) \qquad \text{(P245.8)}$$

$$E_S = 0 \quad \text{(P245.9)}$$

$$E_W = E_C \quad \text{(P245.10)}$$

$$E_I = 0 \quad \text{(P245.11)}$$

The total moisture flux, E_* , is that calculated in component P244 using (P244.125).

c) **Snowfree land with negative surface moisture flux and surface temperature below freeezing.**

In this case there is deposition (negative sublimation) onto the surface and so

$$E_I = E_* \quad (< 0) \quad \text{(P245.12)}$$

$$E_C = 0 \quad \text{(P245.13)}$$

$$E_S = 0 \quad \text{(P245.14)}$$

$$E_W = 0 \quad \text{(P245.15)}$$

(The snow amount is incremented using E_I in component P251 although strictly this is frost rather than snow.)

d) **Land with "shallow snow".**

If the snow lying on the land surface is less than the surface moisture flux integrated over the timestep then the remainder of flux must come from he canopy and/or the soil. "Shallow snow" is therefore taken to mean that $E_* \delta t \geq S \; (\geq 0)$ where E_* is the total surface moisture flux calculated in component P244 using (P244.125) and S is the surface snow amount. In this case the sublimation is given by

$$E_I = S / \delta t \quad \text{(P245.16)}$$

The moisture flux remaining for evaporation from the canopy and/or the soil, E_W , is given by

$$E_W = E_* - E_I \qquad \text{(P245.17)}$$

The canopy evaporation is initially set as

$$E_C = f_A E_W \qquad \text{(P245.18)}$$

where f_A is given by (P243.68) and the soil evaporation is initially set as

$$E_S = (1 - f_A) E_W \qquad \text{(P245.19)}$$

(Note that this expression for the soil evaporation includes only aerodynamic resistance. It is used only infrequently and so this is not likely to lead to serious inaccuracies.) A test is then done to see if the canopy evaporation can be sustained over the timestep. If not, i.e.

if $E_C \delta t > c$, then the canopy evaporation is adjusted and the remaining evaporation is assumed to come from the soil. The algorithm for calculating the adjusted values (denoted by a prime) is:

if $E_C \delta t > c$ then

$$E_S' = (1 - f_A c / (E_C \delta t)) E_W \qquad \text{(P245.20)}$$

$$E_C' = c / \delta t \qquad \text{(P245.21)}$$

else

$E_S' = E_S$ (given by (P245.19))

$E_C' = E_C$ (given by (P245.20))

The reason for the factor $c / (E_C \delta t)$ in (P245.20) is explained in a) above. A further adjustment of the soil evaporation may be necessary if $E_S' \delta t > m$; this is done exactly as in case a) (see (P245.5 and 6)). Finally the total evaporation of surface and subsurface liquid water, E_W , is reset using (P245.7).

e) **Land with sublimation from deep snow or with deposition onto a snow covered surface.**

Physically the case $0 \le E_* \delta t < S$ (i.e. sublimation from deep snow) and the case $E_* \delta t < 0 < S$ (frost being deposited onto a snow covered surface) are distinct but both satisfy $E_* \delta t < S$ and are treated by setting:

$$E_I = E_* \qquad \text{(P245.22)}$$

$$E_C = 0 \qquad \text{(P245.23)}$$

$$E_S = 0 \qquad \text{(P245.24)}$$

$$E_W = 0 \qquad \text{(P245.25)}$$

(Deposition onto a snow-free surface, i.e. the case $E_* \delta t < S = 0$, has already been treated in c).)

After E_C , E_S , E_W and E_I have been set and adjusted if necessary at land points the adjusted total evaporation is set as

$$E_*' = E_W + E_I \qquad \text{(P245.26)}$$

The canopy evaporation, soil evaporation and sublimation are passed out of this subcomponent for use in updating the canopy water content, soil moisture content and surface snow amount respectively in component P25.

The adjustments to the surface moisture fluxes at land points imply that there are further increments to T_* and T_{Lk} and q_{Wk} for $1 \le k \le BL\_LEVELS$. Denoting these by $\delta' X$ the adjusted variables become

$$T_*'^{\,n+1} = T_*^n + \delta T_* + \delta' T_* = T_*^{n+1} + \delta' T_*$$

(P245.101) $T_{Lk}{'}^{n+1} = T_{Lk}^{n} + \delta T_{Lk} + \delta' T_{Lk} = T_{Lk}^{n+1} + \delta' T_{Lk}$

(P245.102) $q_{Wk}{'}^{n+1} = q_{Wk}^{n} + \delta q_{Wk} + \delta' q_{Wk} = q_{Wk}^{n+1} + \delta' q_{Wk}$

(P245.103) where X^{n+1} is the value of X after updating in subcomponent P244 and δX is the increment calculated there.

The method of calculating $\delta' X$ depends on whether Scheme 1 or Scheme 2 for boundary layer mixing is used.

Scheme1

The adjusted surface heat flux, H_*' , is given by

$$H_*' = H_*^{n} - \gamma_1 c_P R K_H(1)(\delta T_{L1} + \delta' T_{L1} - \delta T_* - \delta' T_*)$$
$$= H_* - \gamma_1 c_P R K_H(1)(\delta' T_{L1} - \delta' T_*)$$

(P245.104) So the equation for the total increment to T_* is

$$\delta T_* + \delta' T_* = \delta t A_{S1}(R_{N\downarrow *} - H_*' - L_C E_W' - (L_C + L_F) E_I' - H_S)$$
$$= \delta t A_{S1}(R_{N\downarrow *} - H_* - L_C E_W' - (L_C + L_F) E_I' - H_S)$$
$$+ \delta t A_{S1} \gamma_1 c_P R K_H(1)(\delta' T_{L1} - \delta' T_*)$$

This can be rewritten as

$$A_* \delta' T_{L1} + B_{T*}' \delta' T_* = \delta t A_{S1}(R_{N\downarrow *} - H_* - L_C E_W' - (L_C + L_F) E_I' - H_S) - \delta T_*$$

(P245.105)

where A_{T*} is given by (P244.16) and

$$B_{T*}' = 1 - A_{T*} \qquad \text{(P245.106)}$$

The adjusted flux of sensible heat ($/c_P$) from layer 1 to layer 2 is

$$F_{TL}'(2) = F_{TL}^{n}(2) - \gamma_2 RK_H(2)(\delta T_{L2} + \delta' T_{L2} - \delta' T_{L1})/\Delta z_{1+1/2}$$
$$= F_{TL}(2) - \gamma_2 RK_H(2)(\delta' T_{L2} - \delta' T_{L1})/\Delta z_{1+1/2} \quad \text{(P245.107)}$$

So the equation for the total increment to T_{L1} is

$$\delta T_{L1} + \delta' T_{L1} = (g\delta t/\Delta p_1)\left(F_{TL}'(2) - H_*'/c_P\right)$$
$$= (g\delta t/\Delta p_1)\left(F_{TL}(2) - H_*/c_P\right)$$
$$- (g\delta t/\Delta p_1)\gamma_2 RK_H(2)(\delta' T_{L2} - \delta' T_{L1})/\Delta z_{1+1/2}$$
$$+ (g\delta t/\Delta p_1)\gamma_1 RK_H(1)(\delta' T_{L1} - \delta' T_*$$

which can be rewritten as

$$T_{T1}\delta' T_{L2} + B_{T1}\delta'^{/} T_{L1} + C_{T1}\delta' T_* = 0$$

(P245.108) since $\delta T_{L1} = (g\delta t/\Delta p_1)\left(F_{TL}(2) - H_*/c_P\right)$

A_{T1} , C_{T1} and B_{T1} are given by (P244.28-30) respectively.

Similar equations can be derived for the adjustment increments to T_{Lk} for

$2 \le k \le BL\_LEVELS$ giving

$$\begin{bmatrix} B_{TN} & C_{TN} & & & \\ A_{TN-1} & B_{TN-1} & C_{TN-1} & & \\ \ddots & \ddots & \ddots & 0 & \\ & \ddots & \ddots & \ddots & \\ 0 & A_{T1} & B_{T1} & C_{T1} & \\ & & A_{T*} & B_{T*}' & \end{bmatrix} \begin{bmatrix} (\delta T_{LN}') \\ (\delta T_{LN-1}') \\ : \\ : \\ \delta T_1 \\ \delta' T_* \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ : \\ : \\ 0 \\ (\delta' T_*)_{ex} - \delta T_* \end{bmatrix}$$

(P245.109)

where the matrix elements A_{Tk} , B_{Tk} , C_{Tk} and A_{T*} are as in subcomponent P244 and B_{T1}' is given above by (P245.106). Also

$$(\delta' T_*)_{ex} = \delta t A_{S1} (R_{N\downarrow *} - H_* - L_C E_W' - (L_C + L_F) E_I' - H_S) \qquad \text{(P245.110)}$$

δT_* is calculated in subroutine IMPL_CAL and passed into subroutine SF_EVAP for use in (P245.109). Note that

$$\begin{aligned} (\delta' T_*)_{ex} - \delta T_* &= \delta t A_{S1} (L_C (E_W - E_W') + (L_C + L_F)(E_I - E_I')) \\ &= \delta t A_{S1} (LH - LH') \end{aligned}$$

(P245.111)

where LH is the surface latent heat flux (' after adjustment).

The adjusted flux of moisture from layer 1 to layer 2 is

$$\begin{aligned} F_{qW}'(2) &= F_{qW}^n(2) - \gamma_2 R K_H(2)(\delta q_{W2} + \delta' q_{W2} - \delta q_{W1} - \delta' q_{W1}) / \Delta z_{1+1/2} \\ &= F_{qW}(2) - \gamma_2 R K_H(2)(\delta' q_{W2} - \delta' q_{W1}) / \Delta z_{1+1/2} \end{aligned}$$

(P245.112)

So the equation for the total increment to q_{W1} is

$$\delta q_{W1} + \delta' q_{W1} = (g\delta t/\Delta p_1)\left(F_{qW}'(2) - E_*'\right)$$
$$= (g\delta t/\Delta p_1)\left(F_{qw}(2) - E_*'\right)$$
$$- (g\delta t/\Delta p_1)\gamma_2 RK_H(2)(\delta' q_{W2} - \delta' q_{W1})/\Delta z_{1+1/2}$$

which can be rewritten as

$$C_{Q1}\delta' q_{W2} + B_{Q1}'\delta' q_{W2} = (g\delta t/\Delta p_1)\left(F_{qW}(2) - E_*'\right) - \delta q_{W1} \qquad \text{(P245.113)}$$

where C_{Q1} is given by (P244.47) and

$$B_{Q1}' = 1 - C_{Q1} \qquad \text{(P245.114)}$$

and δq_{W1} is calculated in subroutine IMPL_CAL and passed into subroutine SF_EVAP.

Similar equations result for the adjustment increments to q_{Wk} for $\leq BL\_LEVELS$

giving

$$\begin{bmatrix} B_{QN} & A_{QN} & & \\ C_{QN-1} & B_{QN-1} & A_{QN-1} & \\ \diagdown & \diagdown & \diagdown & 0 \\ & \diagdown & \diagdown & \diagdown \\ 0 & C_{Q2} & B_{Q2} & A_{Q2} \\ & & C_{Q1} & B_{Q1}' \end{bmatrix} \begin{bmatrix} (\delta' q_{WN}) \\ (\delta' q_{WN-1}) \\ : \\ : \\ \delta' q_2 \\ \delta' q_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ : \\ : \\ 0 \\ (\delta' q_{W1})_{ex} - \delta q_{W1} \end{bmatrix}$$

(P245.115)

where the matrix elements are as in subcomponent P244 except for B_{Q1}' which is given by (P245.114). Also

$$(\delta' q_{W1})_{ex} = (g\delta t/\Delta p_1)\left(F_{qW}(2) - E_*'\right) \quad \text{(P245.116)}$$

giving

$$(\delta' q_{W1})_{ex} - \delta q_{W1} = (g\delta t/\Delta p_1)(E_* - E_*') \quad \text{(P245.117)}$$

The solution of these equations by Gaussian elimination would be time consuming so it has been assumed that $\gamma_1 = 1$ and $\gamma_k = 0$ for $2 \le k \le BL\_LEVELS$ in the calculation of the adjustment increments. This considerably simplifies the problem to

$$\delta' q_{W1} = (g\delta t/\Delta p_1)\left(F_{qW}(2) - E_*'\right) - \delta q_{W1} \quad \text{(P245.118)}$$

$$\delta q_{Wk}' = 0 \qquad for \ \ 2 \le k \le BL\_LEVELS \quad \text{(P245.119)}$$

$$\begin{bmatrix} B_{T1}' & C_{T1}' \\ A_{T*}' & B_{T*}' \end{bmatrix} \begin{bmatrix} \delta' T_{L1} \\ \delta' T_* \end{bmatrix} = \begin{bmatrix} 0 \\ (\delta' T_*)_{ex} - \delta T_* \end{bmatrix} \quad \text{(P245.120)}$$

$$A_{T*}' = -\delta t A_{S1} c_P R K_H(1) \quad \text{(P245.121)}$$

$$B_{T*}' = 1 - A_{T*}' \quad \text{(P245.122)}$$

$$C_{T*}' = -(g\delta t/\Delta p_1) R K_H(1) \quad \text{(P245.123)}$$

$$B_{T*}' = 1 - A_{T*}' \quad \text{(P245.124)}$$

(P245.120) can be solved to give

$$\delta' T_* = (1 - C_{T1}')\left((\delta' T_*)_{ex} - \delta T_*\right) / (1 - C_{T1}' - A_{T*}') \quad \text{(P245.125)}$$

$$\delta' T_{L1} = - C_{T1}'\left((\delta' T_*)_{ex} - \delta T_*\right) / (1 - C_{T1}' - A_{T*}') \quad \text{(P245.126)}$$

$$\delta' T_{Lk} = 0 \qquad for \quad 2 \le k \le BL\_LEVELS \qquad \text{(P245.127)}$$

The adjusted fluxes are then

$$F_{qW}'(k) = F_{qW}(k) \qquad for \quad 2 \le k \le BL\_LEVELS \qquad \text{(P245.128)}$$

$$H_*' = H_* + c_P R K_H(1)\left((\delta' T_*)_{ex} - \delta T_*\right) / (1 - C_{T1}' - A_{T*}') \qquad \text{(P245.129)}$$

$$F_{TL}'(k) = F_{TL}(k) \qquad for \quad 2 \le k \le BL\_LEVELS \qquad \text{(P245.130)}$$

The simplified adjustment calculations can sometimes cause q_{W1} to become negative since with $\gamma_2 = 0$, $F_{qW}(2)$ is not adjusted whereas $E_*' \le E_*$. Using (P245.118) it can be seen that this can give a total increment which is negative and large enough to make the final q_{W1} negative. This is undesirable but occurs infrequently (only when the adjustment to E_* is large and the unadjusted flux out of layer 1 is large and positive). If the forward timelevel weighting coefficients were set to the same values as in subcomponent P244 the problem would not arise.

Note that if γ_1 is also set to zero, the adjustment procedure is entirely explicit, and the solution is simplified further giving

$$\delta' T_* = (\delta' T_*)_{ex} - \delta T_*$$

$$\delta' T_{L1} = 0$$

$$H_*' = H_*$$

This has been found to give numerical instabilities and so γ_1 is set to 1 leading to (P245.124,125 and 128).

<u>Scheme 2</u>

The adjusted surface heat flux, H_*' , is given by

$$H_*' = H_* - \gamma_1 c_P R K_H(1)\Big((\delta' T_{L1})_{lm} + (\delta' T_L)_{rm} - \delta' T_*\Big) \qquad \text{(P245.201)}$$

if $N_{rml} \geq 2$, otherwise the fluxes and increments are as in Scheme 1.

So the equation for the total increment to T_* is

$$\begin{aligned} \delta T_* + \delta' T* &= \delta t A_{S1}(R_{N\downarrow *} - H_*' - L_C E_W' - L_C + L_F) E_I' - H_S \\ &= (\delta t A_{S1}(R_{N\downarrow *} - H* - L_C E_W' - (L_C + L_F) E_I' - H_S) \\ &+ \delta t A_{S1} \gamma_1 c_P R K_H(1)\Big((\delta' T_{L1})_{lm} + (\delta' T_L)_{rm} - \delta' T_*\Big) \end{aligned}$$

This can be rewritten as

$$A_{T*}\Big((\delta' T_{L1})_{lm} + (\delta' T_L)_{rm}\Big) + B_{T*}\delta' T_* = (\delta' T_*)_{ex} - \delta T_* \qquad \text{(P245.202)}$$

where A_{T*} is given by (P244.16) and

$$B_{T*}' = 1 - A_{T*} \qquad \text{(P245.203)}$$

$$(\delta' T_*)_{ex} = \delta t A_{S1}(R_{N\downarrow *} - H_* - L_C E_W' - (L_C + L_F) E_I' - H_S) \qquad \text{(P245.204)}$$

The adjusted flux of sensible heat $(/c_P)$ due to local mixing from layer $k - 1$ to

layer k for $2 \leq k \leq N_{rml}$ is

$$F_{TL}^{(lm)}{}'(k) = F_{TL}^{(lm)}(k) - \gamma_k R K_H(k)\Big((\delta' T_{Lk})_{lm} - (\delta' T_{Lk-1})\Big)_{lm} / \Delta_{k-1/2}$$

(P245.205) The equation for the total local mixing increment to T_{Lk} for $2 \leq k \leq N_{rml} - 1$ is

$$(\delta' T_{Lk})_{lm} + (\delta T_{Lk})_{lm} = (g\delta t / \Delta p_k)\Big(F_{TL}^{(lm)}{}'(k+1) - F_{TL}^{(lm)}{}'(k)\Big)$$

(P245.206) which gives, after substitution for the adjusted fluxes from (P245.205)

$$A_{Tk}(\delta'T_{Lk+1})_{lm} + B_{Tk}(\delta'T_{Lk})_{lm} + C_{Tk}(\delta'T_{Lk-1})_{lm}$$
$$= (g\delta t/\Delta p_k)\left(F_{TL}^{(lm)}(k+1) - F_{TL}^{(lm)}(k)\right) - (\delta T_{Lk})_{lm} = 0$$

Similarly for $k = 1$

$$A_{T1}(\delta'T_{L2})_{lm} + B_{T1}(\delta'T_{L1})_{lm} = (g\delta t/\Delta p_k)F_{TL}^{(lm)}(2) - (\delta T_{L1})_{lm} = 0$$

and for $k = N_{rml}$,

$$B_{Tk}(\delta'T_{Lk})_{lm} + C_{Tk}(\delta'T_{Lk-1})_{lm} = -(g\delta t/\Delta p_k)F_{TL}^{(lm)}(k) - (\delta T_{Lk})_{lm} = 0$$

So $(\delta'T_{Lk})_{lm} = 0$ for $1 \le k \le N_{rml}$ if $N_{rml} \ge 2$. (P245.202) therefore becomes

$$A_{T*}(\delta'T_L)_{rm} + B_{T*}'\delta'T_* = (\delta'T_{T*})_{ex} - \delta T_* \qquad \text{(P245.207)}$$

The equation for the total increment due to rapid mixing can be written as

$$A_{Trml}\delta'T_{L,Nrml} + B_{Trml}(\delta'T_L)_{rm} + C_{Trml}\,\delta'T_* = 0$$

(P245.208) Similarly for $k = N_{rml} + 1 < BL\_LEVELS$

$$A_{Tk}\delta'T_{Lk+1} + B_{Tk}\,\delta'T_{Lk} + C_{Tk}\,(\delta'T_L)_{rm} = 0$$

(P245.209) For $k = N_{rml} + 1 < BL\_LEVELS$ (P245.209) still applies with the first term missing. The equations for $\delta'T_{Lk}$ for $N_{rml} + 2 \le k \le BL\_LEVELS$ (only required if $N_{rml} \le BL\_LEVELS - 2$) are the same as those for Scheme 1.

The following matrix equation incorporates (P245.207-209)

$$
\begin{bmatrix}
B_{TN} & & C_{TN} & & & & & \\
& \ddots & & \ddots & & & & \\
& & \ddots & & \ddots & & & \\
& & & \ddots & & \ddots & & \\
& & A_{TN_r+1} & & B_{TN_r+1} & C_{TN_r+1} & & \\
& & & & A_{Trml} & B_{Trml} & C_{Trml} & \\
& & & & & & A_{T*}' & B_{T*}'
\end{bmatrix}
\begin{bmatrix}
\delta' T_{LN} \\ : \\ : \\ \delta' T_{LN_r+1} \\ (\delta' T_L)_{rm} \\ \delta' T_*
\end{bmatrix}
=
\begin{bmatrix}
0 \\ : \\ : \\ 0 \\ 0 \\ (\delta' T_*)_{ex} - \delta T_*
\end{bmatrix}
$$

(P245.210)

Similarly the adjusted increments to q_W are given by the solution to

$$
\begin{bmatrix}
B_{QN} & & A_{QN} & & & & \\
& \ddots & & \ddots & & & \\
& & \ddots & & \ddots & & \\
& & & \ddots & & \ddots & \\
& & C_{QN_r+1} & & B_{QN_r+1} & A_{QN_r+1} & \\
& & & & C_{Qrml} & B'_{Qrml} &
\end{bmatrix}
\begin{bmatrix}
\delta' q_{WN} \\ : \\ : \\ \delta' q_{WN_r+1} \\ (\delta' q_W)_{rm}
\end{bmatrix}
=
\begin{bmatrix}
0 \\ : \\ : \\ 0 \\ (\delta' q_W)_{rm,ex} - (\delta q_W)_{rm}
\end{bmatrix}
$$

(P245.211)

where

$$B'_{Qrml} = 1 - C_{Qrml} \qquad \text{(P245.212)}$$

$$(\delta' q_W)_{rm.ex} = (g\delta t/\Delta p_{rml}) \left(F_{qW}(N_{rml}+1) - E_*'\right)$$

(P254.213) (P245.213) implies

$$(\delta' q_W)_{rm,ex} - (\delta q_W)_{rm} = (g\delta t/\Delta p_{rml}) \; (E_* - E_*') \qquad \text{(P245.214)}$$

There are no local mixing adjustment increments to q_W for $1 \le k \le N_{rml}$ if $N_{rml} \ge 2$ for the same reason as for T_L .

Now, as in Scheme1, assume $\gamma_1 = 1$ and $\gamma_k = 0$ for $1 \le k \le BL\_LEVELS$;

this gives

$$(\delta' q_W)_{rm} = (g\delta t/\Delta p_{rml})\ (E_* - E_*') \tag{P245.215}$$

$$\delta' q_{Wk} = 0 \quad for \quad N_{rml}+1 \le k \le BL\_LEVELS \ if \ N_{rml} \ge 2$$

(P245.216) $(\delta' q_{Wk})_{lm} = 0 \quad for \quad 1 \le k \le N_{rml} \quad if \ N_{rml} \ge 2$

and

$$\begin{bmatrix} B'_{Trml} & C'_{Trml} \\ A'_{T*} & B'_{T*} \end{bmatrix} \begin{bmatrix} (\delta' T)_{rm} \\ \delta' T_* \end{bmatrix} = \begin{bmatrix} 0 \\ (\delta' T_*)_{ex} - \delta T_* \end{bmatrix} \tag{P245.218}$$

where

$$C'_{Trml} = (g\delta t/\Delta p_{rml}) RK_H(1)$$

(P245.219) $B'_{Trml} = 1 - C'_{Trml}$

(P245.220) $A'_{T*} = -\delta t A_{S1} c_P RK_H(1)$

(P245.221) $B'_{T*} = 1 - A'_{T*}$

(P245.222) (P245.218) can be solved to give

$$\delta' T_* = (1 - C'_{Trml})\left((\delta' T_*)_{ex} - \delta T_*\right)/(1 - C'_{Trml} - A'_{T*}) \tag{P245.223}$$

$$(\delta' T_L)_{rm} = -C'_{Trml})\left((\delta' T_*)_{ex} - \delta T_*\right)/(1 - C'_{Trml} - A'_{T*}) \tag{P245.224}$$

$$\delta' T_{Lk} = 0 \quad for \quad N_{rml}+1 \le k \le BL\_LEVELS \tag{P245.225}$$

The surface temperature is updated according to (P245.101) and T_{Lk} and q_{Wk} (for

$for \quad 1 \le k \le N_{rml} \quad if \ N_{rml} \ge 2$) are updated using

$$T'^{n+1}_{Lk} = T^{n+1}_{Lk} + \left(\delta' T_L\right)_{rm}$$

(P245.226) $$q'^{n+1}_{Wk} = q^{n+1}_{Wk} + \left(\delta' q_W\right)_{rm}$$

(P245.227) The adjusted fluxes are given by

$$H'_* = H_* + c_P R K_H(1)\left((\delta' T_*)_{ex} - \delta T_*\right) / (1 - C'_{Trml} - A'_{T*}) \qquad \text{(P245.228)}$$

$$F'_{TL}(k) = F_{TL}(k) \quad for \quad N_{rml}+1 \le k \le BL\_LEVELS \qquad \text{(P245.229)}$$

$$F'_{qW}(k) = F_{qW}(k) \quad for \quad N_{rml}+1 \le k \le BL\_LEVELS \qquad \text{(P245.230)}$$

For $2 \prec k \le N_{rml}$ (P244.294) gives

$$F_{TL'}(k) = F^{(1m)}_{TL}(k) + F_{TL}'(k-1) - F^{(1m)}_{TL}(k-1)$$
$$+ \left((\delta T_L)_{rm} + (\delta' T_L)_{rm}\right) / (g \delta t / \Delta p_{k-1})$$

(P245.231)

So $\delta' F_{TL}(k) \equiv F_{TL}'(k) - F_{TL}(k)$ is given by

$$\delta' F_{TL}(k) = \delta' F_{TL}(k-1) + (\delta' T_L)_{rm} / (g \delta t / \Delta p_{k-1}) \qquad \text{(P245.232)}$$

For $k = 2$, (P244.295) gives

$$\delta' F_{TL}(2) = \delta' H_* / c_P + (\delta' T_L)_{rm} / (g \delta t / \Delta p_1) \qquad \text{(P245.233)}$$

Similar equations apply for $\delta' F_{qW}(k)$ if $1 \prec k \le N_{rml}$ with the surface moisture flux adjustment calculated from

$$\delta' F_{TL}(1) \equiv \delta' E_* - E_* = -(\delta' q)_{rm} / (g \delta t / \Delta p_{rml}) \qquad \text{(P245.234)}$$

which follows from (P245.215).

(iii) <u>The adjustment of the surface temperature increment at sea-ice points</u> For non-land points the canopy evaporation, E_C , and soil evaporation, E_S , are set to zero. At sea points with sea-ice fraction greater than zero the surface temperature of the sea-ice cannot rise above its melting point, T_M . This implies that the gridbox mean surface temperature at sea-ice points must be reduced if it has risen above $T_{*(\max)}$ given by

$$T_{*(\max)} = f_I T_M + (1-f_I) T_{FS} \qquad \text{(P245.29)}$$

This expression follows from substituting the maximum value of $T_{*(I)}$ in (P241.2). The heat flux associated with the melting (weighted with the sea-ice fraction to give the gridbox mean), $H_{*I(T)}$, is given by

$$\begin{aligned} H_{*I(T)} &= f_I (T_{*I} - T_M) / (\delta t A_I) \\ &= (\delta' q_{Wk})_{lm} = 0 \quad for \quad 1 \le k \le N_{rml} \quad if \quad N_{rml} \ge 2 \end{aligned} \qquad \text{(P245.30)}$$

if $T_* > T_{*(MAX)}$, otherwise it is zero.

Also at sea-ice points the sublimation, E_I , is set as

$$E_I = E_{*(I)} \qquad \text{(P245.31)}$$

where $E_{*(I)}$ is the sublimation from the sea-ice weighted with the sea-ice fraction, calculated in subcomponent P244 using (P2440.2). The sublimation is set to zero at sea points with no sea-ice.

The algorithm including the above considerations for non-land points is

$$E_C = 0$$
$$E_S = 0$$

if $\int_I > 0$ then

$$E_I = E_{*(I)} \qquad \text{(see (P245.31))}$$

$$T_{*(MAX)} = f_I T_M + (1-f_I) T_{FS}$$ (see (P245.29))

if $T_* > T_{*MAX)}$ then

$$H_{*I(T)} = (T_* - T_{*(MAX)} / (\delta t A_I)$$ (see (P245.30)

$$T_*' = T_{*(MAX)}$$ (adjusted T_*)

else

$$H_{*I(T)} = 0$$

endif

else

$$E_I = 0$$

$$H_{*I(T)} = 0$$

endif

(iii) <u>The diagnostic calculation of temperature and humidity at 1.5m</u>

After the surface temperature and layer 1 total water content have been updated and possibly adjusted the liquid/frozen water temperature and total water content at the reference height of 1.5m above the surface can be diagnosed if required. This is done using

$$T_L(1.5m) = T_* - g(1.5 + z_{0m} - z_{0h})/c_P + C_{HR}(1.5m)\left(T_{L1} - T_* + g(z_1 + z_{0m} - z_{0h})/c_P\right) \quad (P245.33)$$

$$q_W(1.5m) = q_{W1} + C_{ER}(1.5m)(q_{W1} - Q_{SAT}(T_*, P_*)) \quad (P245.34)$$

These follow from P243.82 and 83, where

$$C_{ER}(1.5m) = \psi(C_{HR}(1.5m) - 1) \quad (P245.35)$$

The temperature, T , humidity, q , cloud liquid water, $q_C^{(L)}$, cloud ice water, $q_C^{(F)}$ and cloud fraction, C , at 1.5m are calculated by calling subroutine LS_CLD for this level immediately after the surface and boundary layer calculations.

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The updated values of the total water content, q , and the liquid/frozen water temperature, T_L , for layers 1 to BL_LEVELS are calculated in subcomponent P244. The updated temperature, T , specific humidity, q , cloud liquid water, $q_C^{(L)}$, cloud ice water, $q_C^{(F)}$, and cloud fraction, C , are calculated by calling subroutine LS_CLD (component P292) for layers 1 to BL_LEVELS immediately after the surface and boundary layer calculations. Another call to subroutine LS_CLD is also made to obtain the 1.5m values of temperature, humidity, cloud water and cloud fraction from the diagnosed T_L and q_W at 1.5ÿm which are output from subcomponent P245.

The surface and boundary layer scheme could be used in a model which does not have prognostic cloud water variables by inputting fields of zeros for cloud liquid water, cloud ice water and cloud fraction. It will then operate as a "dry" scheme and the updated output quantities will already be temperature and specific humidity. In this mode of use calls to subroutine LS_CLD need not and should not be made.