UNIFIED MODEL DOCUMENTATION PAPER NO 24

SUBSURFACE, SURFACE AND BOUNDARY LAYER PROCESSES

by

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Version No 2

31st March 1993

24 March 1995: note that this document does not cover version 3.3a of the B.L. scheme (R.M.L. with new orogrpahic roughness, released with version 3.4 of the Unified Model)

Model Version 3.1

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Modification Record		
Document version	**Author**	**Description...................**

UNIFIED MODEL COMPONENT P24

Subsurface, surface and boundary layer processes

Introduction

Processes at the earth's surface interact with those in the atmosphere on all space and time scales and so a model must represent surface processes for accurate atmospheric predictions. Since most human and other biological activity takes place at or very near the surface, prediction of variables characterising the thermodynamic and hydrological state of the surface is important in its own right. Subsurface thermodynamics and hydrology must also be modelled to predict surface quantities.

The surface exerts its influence on the free atmosphere through the atmospheric boundary layer. This can be from a few tens of metres to 1-2 km deep depending on the stability which determines the intensity and depth of the turbulent transport of momentum, heat and moisture.

This component is divided into a number of subcomponents:

P241 Sea-ice thermodynamics - this calculates the heat flux through sea-ice.

P242 Soil thermodynamics - this calculates the heat flux through the soil at land points. (Subsurface hydrology is treated in component P25 and processes beneath the sea surface are not considered in non-coupled configurations or treated in component P4 in coupled configurations.)

P243 Turbulent surface exchange and boundary layer mixing coefficients - the processes which determine the turbulent fluxes of momentum, heat and moisture at the surface and through the boundary layer are treated here.

P244 The surface and boundary layer equations and their implicit solution - this subcomponent calculates the increments to the surface temperature and to boundary layer temperature, moisture and wind using an implicit numerical scheme.

P245 Adjustment of the surface evaporation and sublimation and the surface temperature increment

- the surface moisture fluxes and hence the boundary layer temperature and moisture increments may need some adjustment if the land surface hydrological stores are too low to sustain the fluxes over a timestep; this component does this adjustment. It also adjusts the sea-ice surface temperature back to the freezing point if melting is taking place.

P246 Boundary layer cloud - the turbulent transport calculated by the boundary layer scheme takes into account the latent heating effects of boundary layer cloud; it does this by using "cloud conserved" variables. This subcomponent calculates the temperature, humidity, cloud water contents and cloud amounts from the updated conserved variables.

Detailed descriptions of these subcomponents are now presented in turn.

P241 Sea-ice thermodynamics (subroutine SICE_HTF)

This component calculates the heat flux through the sea-ice. For land points and for seapoints where the sea-ice fraction, f_I, is zero the flux is set to zero. The sea-ice thermodynamics is treated very simply with a single layer model (see the appendix to Semtner, 1976): at sea points where $f_I \succ 0$ the flux through the ice fraction is assumed to be the product of a thermal conductivity parameter for sea-ice, κ_I, and the temperature gradient across the ice layer. The temperature of the ice layer at its interface with the underlying sea is assumed to be the freezing point of sea-water, T_{FS}, since this is the temperature at which the ice and sea-water can co-exist in equilibrium.

The flux required for calculating the gridbox mean surface temperature increment in component P244 and for use in the sea-ice component of the coupled model (see component P4) is the gridbox mean value and so the local sea-ice flux must be weighted with the sea-ice fraction. The gridbox mean heat flux through the sea-ice, H_I, defined to be positive downwards, is therefore given by

$$H_I = f_I \kappa_I (T_{*I} - T_{FS}) / D_I \qquad \text{(P241.1)}$$

where D_I is the "equivalent thickness" of the sea-ice layer (i.e. the actual thickness plus an additional amount to account for any snow lying on the ice) and T_{*I} is the temperature of the sea-ice at its interface with the atmosphere. f_I and D_I are inputs to the subroutine; in atmosphere-only configurations of the model f_I and D_I are specified from climatology or analysed from observations but in coupled configurations they are predicted by the sea-ice model. T_{*I} is related to the gridbox mean surface temperature, T_*, through the equation

$$T_* = f_I T_{*I} + (1 - f_I) T_{FS} \qquad \text{(P241.2)}$$

where it is assumed that the surface temperature of the sea-water fraction of the gridbox is T_{FS} (only when $f_I \succ 0$). Substituting for $f_I T_{*I}$ in (P241.1) from (P242.2) gives

$$H_I = \kappa_I (T_* - T_{FS}) / D_I \qquad \text{(P241.3)}$$

κ_I is set to 2.09 $\mathrm{W\,m^{-1}\,K^{-1}}$ and $T_{FS} = T_M - 1.8\mathrm{K}$ where T_M is the melting point of ice (= the freezing point of pure water). H_I is evaluated using the timelevel n value of the surface temperature i.e. T_*^n.

Reference

Semtner,A.J., 1976: A model for the thermodynamic growth of sea ice in numerical investigations of climate. *J. Phys. Oceanog.*, **6**, 379-389.

P242 Soil thermodynamics (subroutine SOIL_HTF)

(i) Introduction

Heat transport through the soil is modelled with a multilayer scheme. The main concern when choosing the number of soil layers and the values of the arbitrary parameters of the scheme (such as the layer thicknesses) is to ensure that the (complex) ratio of the finite difference scheme's solution for the surface temperature to the analytic solution has amplitude close to unity and phase close to zero for the range of surface forcing frequencies occurring in nature. It has been found that a four layer scheme with appropriate values of parameters gives a good amplitude and phase response for periods of surface forcing between half a day and a year. The description of the scheme below is for a general number of soil layers,
N_s = DS_LEVELS + 1, until the point where parameters need to be specified to fully document the model.

This component updates the "deep" soil temperatures, i.e. all except the top soil layer temperature, and outputs the heat flux between the top two soil layers for use in the implicit calculation of the increments to the top soil temperature and boundary layer variables (in component P244). Note that there are DS_LEVELS deep soil temperatures at each land gridpoint. The increments to the deep soil temperatures and the heat fluxes between all the soil layers are calculated "explicitly", i.e. in terms of timelevel n variables.

(ii) The multilayer soil thermodynamics model

The continuous equations for the soil heat flux, H_s, and rate of change of T_s, the soil temperature are

$$H_s = -\lambda_s \partial T_s / \partial z \qquad \text{(P242.1)}$$

$$C_s \partial T_s / \partial t = -\partial H_s / \partial z \qquad \text{(P242.2)}$$

where λ_s is the thermal conductivity of the soil (units: $\mathrm{W\,m^{-1}\,K^{-1}}$) and C_s is the volumetric specific heat (or heat capacity) of the soil (units: $\mathrm{J\,m^{-3}\,K^{-1}}$). Both z and H_s are defined to be positive downwards.

At the top boundary of the soil model the downward heat flux is the sum of the net radiative flux and the turbulent sensible and latent heat fluxes at the surface, $R_{N\downarrow *} - H_* - LE_*$, (the turbulent fluxes are defined in component P244; they are positive upwards). At the bottom boundary the heat flux is assumed to be zero. An alternative lower boundary condition would be to set the temperature at the bottom of the model to a climatological value. This is unacceptable when using the model for climate simulations because it effectively introduces an infinite source/sink of heat in the soil.

Discretizing with respect to z and combining (P242.1) and (P242.2) we obtain the following equations for the rate of change of the deep soil temperatures (numbering from the top layer downwards):

$$\frac{\partial T_{Sr}}{\partial t} = A_{Sr}(T_{Sr} - T_{Sr-1}) + B_{Sr}(T_{Sr+1} - T_{Sr}) \qquad 2 \le r \le N_s - 1 \qquad \text{(P242.3)}$$

$$\frac{\partial T_{Sr}}{\partial t} = A_{Sr}(T_{Sr} - T_{Sr-1}) \qquad r = N_S \qquad \text{(P242.4)}$$

The coefficients A_{Sr} and B_{Sr} in (P242.3) and (P242.4) are defined in terms of the soil layer thicknesses Δz_r and the soil diffusivity $\kappa_S = \lambda_S/C_S$ (assumed uniform with depth) by

$$A_{S1} = 1/C_S\Delta z \qquad \text{(P242.5)}$$

$$A_{Sr} = -\frac{2\kappa_S}{\Delta z_r(\Delta z_r + \Delta z_{r-1})} \qquad 2 \le r \le N_S \qquad \text{(P242.6)}$$

$$B_{Sr} = \frac{2\kappa_S}{\Delta z_r(\Delta z_{r+1} + \Delta z_r)} \qquad 1 \le r \le N_S - 1 \qquad \text{(P242.7)}$$

The heat flux between soil layers r and $r+1$, $H_S(r,r+1)$, is given by

$$\begin{aligned} H_S(r,r+1) &= -2\lambda_S(T_{Sr+1} - T_{Sr})/(\Delta z_{r+1} + \Delta z_r) \\ &= -(\zeta_r B_{Sr}/A_{S1})(T_{Sr+1} - T_{Sr}) \\ &= (\zeta_{r+1} A_{Sr+1}/A_{S1})(T_{Sr+1} - T_{Sr}) \qquad 1 \le r \le N_S - 1 \end{aligned}$$

(P242.8)

where $\zeta_r = \Delta z_r/\Delta z_1$ (P242.9)

For $r = 2$ to $N_S - 1$ the heat flux given by (P242.8) is an optional diagnostic output but for $r = 1$ the flux must be output from this subcomponent since it is required by subcomponent P244 to calculate the increment to T_{S1}.

Δz_1 is chosen to be the e-folding depth of a temperature wave of characteristic frequency ω_1 which satisfies the continuous equations (P242.1) and (P242.2); it can be shown that this implies

$$\Delta z_1 = (2\lambda_S/\omega_1 C_S)^{1/2} \qquad \text{(P242.10)}$$

With definitions (P242.9) and (P242.10), the coefficients A_{Sr} and B_{Sr} defined by (P242.5)-(P242.7) can be rewritten as

$$A_{S1} = (\omega_1/2)^{1/2}/\gamma_S \qquad \text{(P242.11)}$$

$$A_{Sr} = \frac{-\omega_1}{\zeta_r(\zeta_r+\zeta_{r-1})} \qquad 2 \le r \le N_S \qquad \text{(P242.12)}$$

$$B_{Sr} = \frac{\omega_1}{\zeta_r(\zeta_{r+1}+\zeta_r)} \qquad 1 \le r \le N_S-1 \qquad \text{(P242.13)}$$

where γ_S is the thermal inertia defined as

$$\gamma_S = (\lambda_S C_S)^{1/2} \qquad \text{(P242.14)}$$

The amplitude and phase responses of the model described above to surface forcing of frequency ω have been investigated. It can be shown that A, the ratio of the multilayer model's surface temperature response to that of the analytic solution to the continuous equations (P242.1) and (P242.2), is a function of the normalised frequency ω/ω_1 and the normalised layer thicknesses ζ_r but is independent of λ_S and C_S (if, as assumed, these parameters are uniform with depth). The ζ_r determine the shape of the response curves and the relative range of frequencies over which $|A| \simeq 1$ and arg $(A) \simeq 0$ and ω_1 determines the absolute position of the response curves on the frequency axis. Four is the minimum number of soil layers required to give a good amplitude and phase response to forcing periods in the range half a day to a year. This range includes the diurnal, seasonal and annual forcing periods in nature. The parameters C_S and λ_S are climatologically prescribed, geographically varying quantities depending on the soil type. Documentation Paper No. 70 describes their derivation from the Wilson and Henderson-Sellers (1985) global archive of land cover and soils data. The other parameters are the same for all gridpoints and are set to

$$\begin{aligned}
N_S &= \text{DS_LEVELS} + 1 = 4, \\
\omega_1 &= 3.55088\text{x } 10^{-4}\ s^{-1}, \\
\zeta_1 &= 1 \text{ (by definition P242.9)}, \\
\zeta_2 &= 3.908, \\
\zeta_3 &= 14.05, \\
\zeta_4 &= 44.65.
\end{aligned} \qquad \text{(P242.15)}$$

The values of the ζ_r are chosen to give the best amplitude and phase response. If the number of soil layers is changed these parameters must be reset.

(iii) <u>The representation of snow insulation</u>

Snow lying on the ground insulates the soil below and for a correct prediction of the surface temperature this effect should be represented. A fully satisfactory scheme may have to include a multilayer snow thermodynamics model sitting on top of the soil model. Such a snow model would be more complicated than that for the soil because the depth of the snow is variable. However, in the scheme currently used snow insulating effects are represented very simply by reducing the thermal conductivity between the top two subsurface layers (no longer strictly soil layers when the snow depth is non-zero).

The modified conductivity, λ', is calculated by assuming that the top subsurface level is still at a depth $\Delta z_1/2$ below the surface (see figure 1). This level is within the snow layer if its thickness, Δz_{SNOW}, is greater than $\Delta z_1/2$, otherwise it is still within the soil and no change to the conductivity is made, i.e. $\lambda' = \lambda_S$. In the former case the conductivities of snow and soil are combined under the assumption that the snow and soil are "in series" so

$$\frac{[(\Delta z_1 + \Delta z_2)/2 + \Delta z_{SNOW}]}{\lambda'} = \frac{\Delta z_{SNOW} - \Delta z_1/2}{\lambda_{SNOW}} + \frac{(\Delta z_1 + \Delta z_2/2)}{\lambda_S} \qquad \text{(P242.16)}$$

where Δz_{SNOW} is the conductivity of snow.

The modified heat flux between soil layers 1 and 2 is

$$H_S'(1,2) = -\lambda'(T_{S2} - T_{S1})/[(\Delta z_1 + \Delta z_2)/2 + \Delta z_{SNOW}] \qquad \text{(P242.17)}$$

Substituting for λ' from (P242.16) in (P242.17) we obtain

$$H_S'(1,2) = H_S(1,2)\Gamma_{SNOW} \qquad \text{(P242.18)}$$

where $H_S(1,2)$ is the heat flux as calculated using (P242.8) with $r = 1$.

Γ_{SNOW} is given by

$$\Gamma_{SNOW} = \begin{cases} \dfrac{(1+\zeta_2)}{\left((2\Delta z_{SNOW}/\Delta z_1)+(1+\zeta_2)\right)} & \text{for } 2\Delta z_{SNOW}/\Delta z_1 \le 1 \\ \dfrac{(1+\zeta_2)}{\left((\lambda_S/\lambda_{SNOW})(2\Delta z_{SNOW}/\Delta z_1 - 1)+(2+\zeta_2)\right)} & \text{for } 2\Delta z_{SNOW}/\Delta z_1 \succ 1 \end{cases}$$

(P242.19)

The coefficients A_{S2} and B_{S1}, which are related to $H_S(1,2)$ via (P242.8) with $r = 1$, are also replaced by

$$A_{S2}' = A_{S2}\Gamma_{SNOW} \qquad \text{(P242.20)}$$

$$B_{S1}' = B_{S1}\Gamma_{SNOW} \qquad \text{(P242.21)}$$

The thickness of the snow layer, Δz_{SNOW}, is given in terms of the mass of snow per unit area, S, by

$$\Delta z_{SNOW} = S/\rho_{SNOW} \qquad \text{(P242.22)}$$

where the value of the density of snow, ρ_{SNOW} is taken to be 250 kg m^{-3}. The thermal conductivity of snow, λ_{SNOW}, is assumed to have the value $0.265 \text{ W m}^{-1} \text{ K}^{-1}$.

For points classified as land-ice in the Wilson and Henderson-Sellers (1985) archive the snow areal density, S, is initialised to $5\text{x}10^4 \text{ kg m}^{-2}$. This is done to ensure that such points never become snow or ice free. At permanent land-ice points the land surface parameters, in particular the thermal conductivity, are already set to values appropriate for the snow and ice covered surface and so no further snow insulation factor should be used. Therefore Γ_{SNOW} is set to 1 if $S \succ 5\text{x}10^3 \text{ kg m}^{-2}$.

Reference

Wilson,M.F. and Henderson-Sellers,A., 1985: A global archive of land cover and soils data for use in general circulation models. *J. Clim.*, **5**, 119-143.

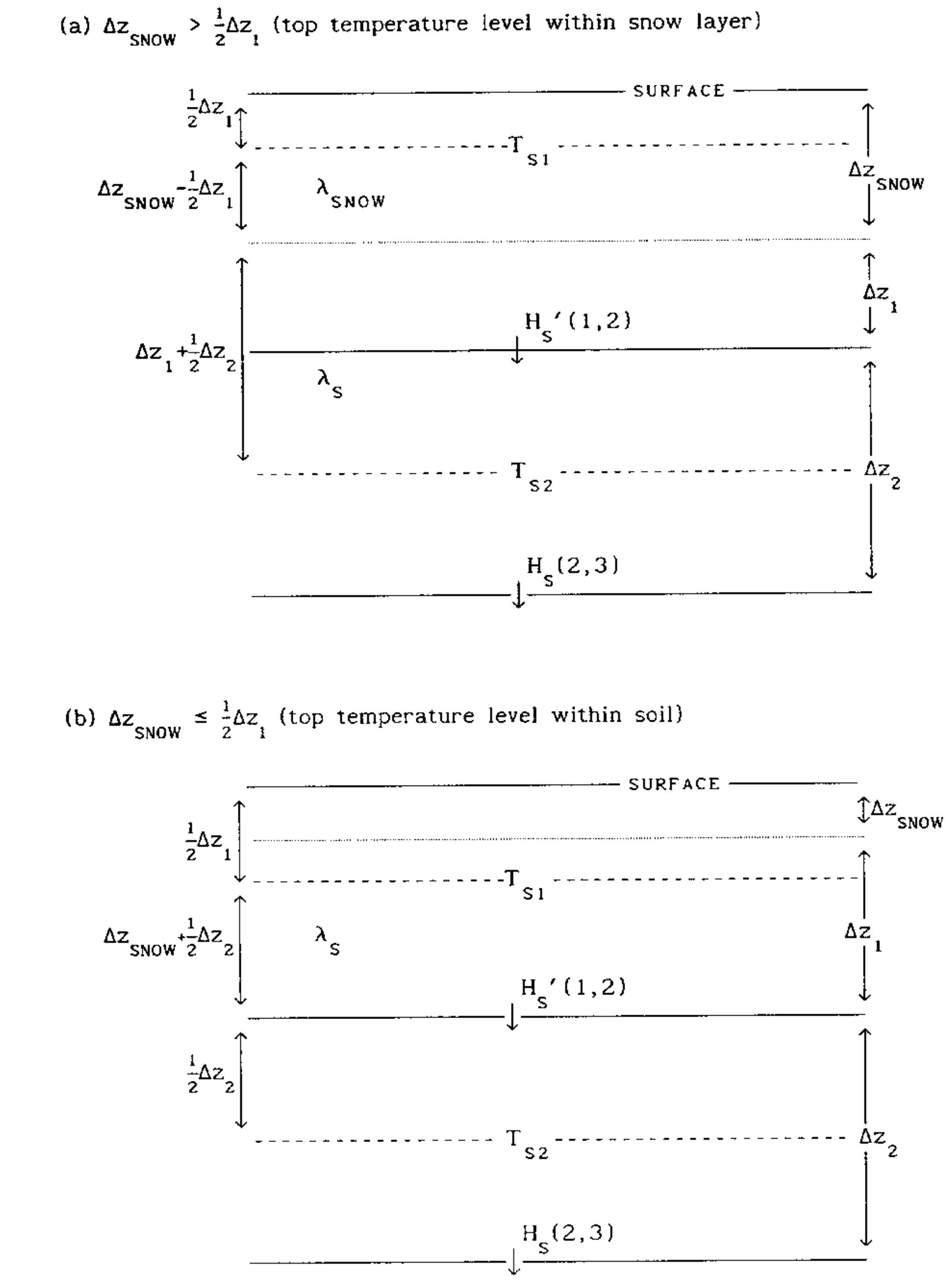

FIGURE 1. The configuration of subsurface temperature levels and layer interfaces when there is lying snow. Solid horizontal lines are layer interfaces, dashed lines are temperature levels and the dotted line is the snow/soil interface. (Schematic - thicknesses not to scale)

P243 Turbulent surface exchange and boundary layer mixing coefficients

(i) Introduction

It is standard practice to represent the mean vertical surface turbulent flux, F_{x^*} , of any conservative quantity X by

$$F_{x^*}/\rho_* \equiv (\overline{w'X'})_* = -c_x|v_1 - v_0|(X_1 - X_0)$$

(P243.1)

where $(\overline{w'X'})$ defines F_{x^*} in conventional turbulence notation (i.e. in terms of the surface eddy covariance of X and the vertical velocity component w,

ρ_* is the atmospheric density at the surface,

v_1 is the mean horizontal wind at the lowest model level,

v_0 is the velocity of the surface (identically zero for land points but equal to the surface ocean current at sea points),

X_1 and X_0 are the values of X at model level 1 and at the surface respectively,

and c_X is the turbulent surface exchange (or bulk transfer) coefficient, which in general is a function of atmospheric stability, surface roughness and other parameters characterising the physical and physiological state of the surface and any vegetation. The bottom model level is assumed to be within the "surface flux" layer (typically a few tens of metres in depth) for the bulk transfer specification (P243.1) to be a good approximation.

The unified model has two schemes available for calculating the turbulent fluxes and increments due to turbulent mixing above the surface. The first (Scheme 1, selected by setting *IF definition A03_1B) is of the standard "local mixing" type, i.e. the turbulent flux, F_X , of a conservative quantity X is parametrized using a first-order turbulence closure

$$F_x/\rho \equiv \overline{w'X'} = -\mathrm{K}_X\ \partial X/\partial z \qquad \text{(P243.2)}$$

where ρ is the atmospheric density and K_X is the turbulent mixing coefficient for X which is in general a function of a mixing length, the local wind shear and atmospheric stability. The functional dependence is specified empirically in the model as described below. The rate of change of the quantity X due to turbulent mixing is then

$$(\partial X/\partial t)_{TM} = g\partial F_X/\partial p) \qquad \text{(P243.3)}$$

An alternative scheme (Scheme 2, selected by setting *IF definition A03_2C) allows non-local mixing of heat and moisture in unstable conditions where the boundary layer is more than one model layer deep. This was developed because of the following potential deficiencies of Scheme 1:

(i) In unstable, rapidly mixing, regions the fluxes are not in fact closely related to local gradients - the eddies or plumes which are doing the mixing have large vertical extent and correlation;

(ii) Forming flux divergences over relatively thin model layers (particularly the lowest model layer) causes numerical problems when the timestep is large and the turbulent mixing coefficients are large (as they are in unstable boundary layers). The implicit numerical scheme (see section P245 below) prevents numerical instability but sometimes at the cost of accuracy; (iii) The local values of stability, on which the K_X's depend, are partly determined by other parts of the model, particularly the convection scheme. The current convection scheme has a tendency to overstabilise the lower boundary layer and hence it can switch off turbulent mixing based entirely on local gradients.

Within the boundary layer Scheme 2 uniformly distributes the heating and moistening resulting from the divergence of the fluxes between the surface and the top of the boundary layer (with the surface fluxes given by (P243.1) and the top-of-boundary-layer fluxes given by (P243.2)). This implies profiles of non-local fluxes, $F_X^{(NL)}$, which are linear with respect to pressure:

$$F_X^{(NL)}(p) = \frac{\left((p_{top} - p)F_{x^*} + (p - p_*)F_{X(top)}\right)}{(p_{top} - P_*)} \qquad \text{(P243.301)}$$

The uniform increments applied to all the model layers within the mixing layer will not alter the shape of the profiles within this layer. This is not very realistic so Scheme 2 also assumes that there is local mixing between the model layers *within* the mixing layer effected by fluxes given by (P243.2). The total flux at a given model layer interface within the mixing layer is then the sum of a non-local flux given by (P243.301) and a local flux given by (P243.2). Further details of both schemes are given in later sections.

The fluxes F_X are assumed to be identically zero at the top of layer BL_LEVELS and above. BL_LEVELS is an integer parameter of the model set to a value sufficiently large that all physically realistic boundary layers and any entrainment at their tops are contained within the bottom BL_LEVELS model layers.

Because of the diffusive nature of the equations which result from either of the schemes, the numerical time-stepping scheme has to be chosen carefully to ensure numerical stability of the solution for reasonably large timesteps. This is done by adopting an implicit scheme described in the documentation for component P244 below. An implication of this is that the turbulent surface exchange and boundary layer mixing coefficients for all layer interfaces have to be calculated, in terms of timelevel n variables, in a preliminary step before the increments due to turbulent mixing can be found. It is the function of component P243 to calculate these coefficients.

<u>(ii) Horizontal interpolation when "staggered grids" are used.</u>

When using this scheme in a model with horizontal wind components and surface ocean currents stored at different positions to surface pressure, temperature and water content variables, the winds and currents are interpolated to the p-grid for use in calculating the surface exchange and boundary layer mixing coefficients. This is done by using the subroutine UV_TO_P. The interpolated wind field on the p-grid is denoted by $\hat{v}$. In the current version of the model, after the surface exchange and boundary layer mixing coefficients for momentum have been calculated on the p-grid they are interpolated to the uv-grid for use in calculating the fluxes of momentum and increments to the wind components. This process is done by using subroutine P_TO_UV. An overbar denotes a quantity interpolated from the p-grid to the uv-grid.

This can be summarised symbolically by

$$\mathrm{K}_X = \mathrm{K}_X\ (\hat{v}, T, q\ \ldots)$$

$$c_X|\hat{v}_1 - \hat{v}_0| = c_x\ (\hat{v}_1, \hat{v}_0, T, q\ldots.)|\hat{v}_1 - \hat{v}_0| \qquad \text{(P243.4)}$$

on the p-grid and

$$\underline{\tau} = -F_v = \overline{\rho \mathrm{K}_v} \partial v / \partial z$$

$$\underline{\tau}_* = -F_{V*} = \overline{\rho_* c_D |\hat{v}_1 - \hat{v}_0| (v_1 - v_0)} \qquad \text{(P243.5)}$$

on the uv-grid.

There is an alternative procedure for dealing with staggered grids which enables all calls to the interpolation subroutines to be made outside the plug-compatible surface and boundary layer processes code. The atmospheric wind field, v, and the ocean surface current field, v_0 would be interpolated to the p-grid in a control-level subroutine and passed into the plug-compatible physics code. The calculations summarised by (P243.4) would remain unchanged. However, the turbulent stress, $\underline{\tau}$, would also be calculated on the p-grid. The mass-weighted increments to $\hat{v}$ would be calculated as $\Delta p.\delta\hat{v} = -g\ \delta\ t.\Delta\underline{\tau}$ and passed out of the plug-compatible code. In the control-level code the increments to v would be calculated on the uv-grid as

$$\delta v = \frac{\overline{\Delta p\ \delta\hat{v}}}{\overline{\Delta p}} \qquad \text{(P243.5')}$$

and v updated by adding this increment. The interpolation of mass-weighted $\hat{v}$ increments to the uv-grid would ensure that the increment to **v** is consistent with the interpolated stress, $\overline{\underline{\tau}}$, because

$$\overline{\Delta p}.\delta v = \overline{\Delta p\ \delta\hat{v}} = -g\ \delta\ t.\Delta\overline{\underline{\tau}} \qquad \text{(P243.5a')}$$

This method has not been implemented or tested in the model so far; it has the potentially major disadvantage of using a stress, $\overline{\underline{\tau}}$, on the uv-grid which is not given in terms of the velocity gradient $\Delta v / \Delta z$ on that grid. Indeed there may be circumstances when the interpolated stress would imply up-gradient transport.

It is worth noting here that p-gridboxes are entirely land or entirely sea or equivalently that the model coastline goes through points on the uv-grid. Thus the value of $\hat{u}$ or $\hat{v}$ at a given point on the p-grid is formed from either all sea values, all land values, a combination of land and coastal values, or a combination of sea and coastal values; sea and land values are never combined in the UV_TO_P interpolation. In contrast the P_TO_UV interpolation can combine sea and land values to obtain coastal values.

<u>(iii) The surface fluxes of momentum, heat and moisture</u>

If X in (P243.1) is set to the vector horizontal wind, v, the surface turbulent flux of momentum is obtained. Conventionally the surface stress, $\underline{\tau}_*$, is defined to be the downward momentum flux at the surface and therefore equal to $-F_{V*}$. So

$$\underline{\tau}_* / \rho_* = -(\overline{w'\ v'})_* = c_D |v_1 - v_0| (v_1 - v_0) \qquad \text{(P243..6)}$$

where c_D is the drag coefficient. For land points v_0 is identically zero but for sea points (including those with sea-ice) v_0 is the ocean surface current input to component P24. In configurations without coupling to an ocean model these quantities are climatological or analysed values but they are predicted when the atmosphere and ocean models are coupled. The so-called **surface friction velocity**, v_S , used as a scaling parameter in the surface layer is defined by

$$v_s = (\underline{\tau}_* / \rho_*) \; / \; |\underline{\tau}_* \; / \; \rho_*|^{1/2} \qquad \text{(P243.7)}$$

So, using (P243.6),

$$v_S = c_d^{1/2}(v_1 - v_0) \qquad \text{(P243.8)}$$

The surface and boundary layer scheme uses thermodynamic and water content variables conserved during the formation and evaporation of cloud, (referred to as "cloud-conserved" variables) (Smith, 1990). These variables are the liquid/frozen water temperature, T_L , defined by

$$T_L = T - \frac{L_C}{c_P} q_C^{(L)} - \frac{(L_C + L_F)}{c_P} q_C^{(F)} \qquad \text{(P243.9)}$$

and the total water content, q_W , defined by

$$q_W = q + q_C^{(L)} + q_C^{(F)} \qquad \text{(P243.10)}$$

where $q_C^{(L)}$ and $q_C^{(F)}$ are the cloud liquid and frozen water contents respectively.

The surface fluxes of sensible heat, H_* , and moisture, E_* , are obtained by putting X equal to the liquid/frozen water static energy, $s_L = c_P T_L + gz$, and the total water content, q_W , respectively in (P243.1):

$$H_*/(c_p \rho_*) = F_{TL*}/\rho_* = (\overline{w'T'_L})_* = -c_H|v_1 - v_0|(T_{L1} - T_* + (g/c_P)z_B) \qquad \text{(P243.11)}$$

$$E_*/\rho_* = F_{qW*}/\rho_* = (\overline{w'q_W'})_* = -\psi c_H|v_1 - v_0|\left(q_{W1} - q_{SAT}(T_*, P_*)\right) \qquad \text{(P243.12)}$$

z_B is the height of the bottom model level above the surface on which $T = T_*$ (see Appendix A for the method of calculating the heights of the model levels above the model's surface). T_* and p_* are the surface temperature and pressure respectively. In (P243.12) any deviation of E_* from its "potential" value (calculated by assuming $q_* = q_{SAT}(T_*, p_*)$), is assumed to be taken account of in the specification of the factor ψ (discussed below). Surface layer scaling parameters for temperature, T_S , and for moisture, v_S , (analogous to v in (P243.7)) can be defined:

$$T_S = -H_*/(\rho_* v_S\ c_P) = \left(c_H/c_D^{1/2}\right)\left(T_{L1} - T_* + (g/c_P)z_B\right) \qquad \text{(P243.13)}$$

$$q_s = -E_*/(\rho_* v_S) = \psi\left(c_H/c_D^{1/2}\right)\left(q_{W1} - q_{SAT}(T_*, p_*)\right) \qquad \text{(P243.14)}$$

where $v_s = |v_S|$.

A **surface buoyancy flux**, F_{B^*} , can be defined as,

$$\frac{F_{B^*}}{\rho_*} = \frac{g}{T_{V1}}\left(\overline{w'\ T'_v}\right) \qquad \text{(P243.15)}$$

where T_V is the **virtual temperature** defined by

$$T_V = T\left(1 + (\epsilon^{-1}-1)q - q_C\right) \qquad \text{(P243.16)}$$

ϵ is the ratio of the molecular weight of water vapour to that of dry air (i.e.

$\epsilon = M_V/M_A = 0.62198$) and $q_C = q_C^{(L)} + q_C^{(F)}$. The surface buoyancy flux can be written in terms of the fluxes of cloud-conserved variables:

$$F_{B*}/\rho_* = g\left(\tilde{\beta}_{T1}\ (\overline{w'T_L'})_* + \tilde{\beta}_{Q1}\ (\overline{w'q_W'})_*\right) \qquad \text{(P243.17)}$$

The buoyancy parameters $\tilde{\beta}_T$ and $\tilde{\beta}_Q$ in (P243.17) are evaluated using model level 1 variables (hence the subscript 1) and defined by

$$\tilde{\beta}_T = \beta_T - \alpha_L\ \beta_C$$

(P243.18)

$$\tilde{\beta}_Q = \beta_Q - \alpha_L\ \beta_C$$

where

$$\beta_T = \frac{1}{T}\ ,\ \beta_Q = \frac{(\epsilon^{-1}-1)}{\left(1+(\epsilon^{-1}-1)q-q_c\right)}\ ,\ \beta_c = Ca_L\left[\frac{L}{c_p}\ \beta_T - \frac{\beta_Q}{(1-\epsilon)}\right] \qquad \text{(P243.19)}$$

$$\alpha_L = \left.\frac{\partial q_{SAT}}{\partial_T}\right|_{T=T_L} = \frac{\epsilon L q_{SAT}\ (T_L, p)}{RT_L^2} \qquad \text{(P243.20)}$$

$$a_l = 1/(1+L\alpha_L/c_p) \qquad \text{(P243.21)}$$

and C is the cloud fraction. The latent heat L is set to L_C or $L_C + L_F$ according to

whether $T_L \succ T_M$ or $T_L \le T_M$ respectively.

The derivation of (P243.17) rests on the hypothesis that the vertical flux of cloud water is given by

$$\overline{w'q_C'} = Ca_L\left(\overline{w'q_W'} - \alpha_L\,\overline{w'T_L'}\right) \qquad \text{(P243.22)}$$

The r.h.s. of (P243.22) reduces to zero when there is no cloud, as it should, and for complete cloud cover it becomes the expression derived from the relationship between q_C , q_W and T_L in saturated air.
Using (P243.17), (P243.11) and (P243.12) we can write

$$F_{B*} = g\left(\tilde{\beta}_{T1}\,(H_*/c_P) + \tilde{\beta}_{Q1}\,E_*\right) \qquad \text{(P243.23)}$$

or alternatively

$$F_{B*}/\rho_* = -c_H|v_1 - v_0|\,\Delta B \qquad \text{(P243.24)}$$

where the difference in buoyancy between model level 1 and the surface, ΔB , is given by

$$\Delta B = g\left[\tilde{\beta}_{T1}\left(T_{L1} - T_* + (g/c_P)z_B\right) + \tilde{\beta}_{Q1}\,\psi\left(q_{W1} - q_{SAT}(T_*,p_*)\right)\right] \qquad \text{(P243.25)}$$

A surface layer buoyancy scaling factor, B_S , can also be defined in terms of the surface buoyancy flux and the friction velocity:

$$B_S = -F_{B*}/(\rho_* v_S)$$

(P243.26)

$$= g\left(\tilde{\beta}_{T1}\,T_S + \tilde{\beta}_{Q1}\,q_s\right)$$

(P243.27)

$$= \left(c_H / c_D^{1/2}\right)\Delta B$$

(P243.28)

Finally a length scale, L_S , called the **Monin-Obukhov length,** can be defined

$$L_S = v_S^2 / (kB_S) \qquad \text{(P243.29)}$$

$$= -\rho_* v_s^3 / \left(kg\left[\tilde{\beta}_{T1}\,(h_*/c_P) + \tilde{\beta}_{Q1}E_*\right]\right)$$

(P243.30)

where $k = 0.4$ is the von Kármán constant.

(iv) The bulk transfer coefficients for momentum, heat and moisture

The previous section gave the surface fluxes in terms of the bulk transfer coefficients c_D and c_H . The dependence of these quantities on the surface layer stability and surface parameters is now discussed. The Monin-Obukhov similarity hypothesis (strictly for a fully turbulent surface layer under stationary and horizontally homogeneous conditions) is the most widely accepted approach for relating surface layer gradients of wind, temperature and moisture to the corresponding surface turbulent fluxes. This gives

$$\frac{\partial v}{\partial z} = \frac{v_s}{kz} \phi_m (z/L_s) \qquad \text{(P243.31)}$$

$$\frac{\partial T_L}{\partial z} + \frac{g}{c_P} = \frac{T_s}{kz} \phi_h (z/L_s) \qquad \text{(P243.32)}$$

$$\frac{\partial q_w}{\partial z} = \frac{q_s}{kz} \phi_h (z/L_s) \qquad \text{(P243.33)}$$

In (P243.31)-(P243.33) v_s, T_s, q_s and L_s are the scaling parameters defined by (P243.7&8), (P243.13), (P243.14) and (P243.29) respectively. ϕ_x is a **universal similarity function** of z/L_s only which in principle may be different for each transferable quantity X and which has to be determined from analysis of surface layer data. However, it has been assumed that the similarity functions for sensible heat and moisture are the same. The lower boundary conditions for (P243.31-33) are respectively

$$v = v_0 \ (= 0 \ \textit{for land points}) \quad at \ z = z_{0m} \qquad \text{(P243.31bc)}$$

$$T_L = T_* \qquad at \ z = z_{0h} \qquad \text{(P243.32bc)}$$

$$q_w = q_* \qquad at \ z = z_{0h} \qquad \text{(P243.33bc)}$$

z_{0m} and z_{0h} are the **surface roughness lengths** for momentum and sensible heat respectively. The roughness length for moisture has been assumed to be equal to that of heat but in principle it may be different. The roughness lengths have to be determined for each surface type from surface layer data. Appendix B gives the values or sources of data for z_{0m} and z_{0h} used in the model. It is convenient to define the model's height coordinate origin at the height where $v=v_0$. This is done by making the transformation $z' = z - z_{0m}$. In terms of this new coordinate (P243.31-33) and their respective boundary conditions become

$$\frac{\partial v}{\partial z'} = \frac{v_s}{k\,(z' + z_{0m})}\,\phi_m\left(\frac{z' + z_{0m}}{L_s}\right) \qquad \text{(P243.31')}$$

$$\frac{\partial T_L}{\partial z'} + \frac{g}{c_P} = \frac{T_s}{k\,(z' + z_{0m})}\,\phi_h\left(\frac{z' + z_{0m}}{L_s}\right) \qquad \text{(P243.32')}$$

$$\frac{\partial q_w}{\partial z'} = \frac{q_s}{k\,(z' + z_{0m})}\,\phi_h\left(\frac{z' + z_{0m}}{L_s}\right) \qquad \text{(P243.33')}$$

with

$$v = v_0 \qquad at\ z' = 0 \qquad \text{(P243.31bc')}$$

$$T_L = T_* \qquad at\ z' = z_{0h} - z_{0m} \qquad \text{(P243.32bc')}$$

$$q_w = q_* \qquad at\ z' = z_{0h} - z_{0m} \qquad \text{(P243.33bc')}$$

Note that if $z_{0m} > z_{0h}$ (as it generally is, particularly if the form drag effects of mountains and obstacles are parametrized using effective roughness lengths) then the surface temperature and humidity are defined at a level *below* the model's surface at $z' = 0$

(P243.31') can be integrated from the model surface at $z' = 0$ to the bottom model level at $z' = z_1$ (or equivalently (P243.31) from $z = z_{0m}$ to $z = z_{0m} + z_1$) to obtain

$$k\,(v_1 - v_0) = v_s \int_0^{z_1} \frac{\phi_m\left((z' + z_{0m}) / L_s\right)}{\left(z' + z_{0m}\right)}\,dz'$$

$$= v_s \int_{\varsigma_{0m}}^{\varsigma_1} \frac{\phi_m\,(\varsigma')}{\varsigma'}\,d\varsigma' \equiv v_s\,\Phi_m\,(\varsigma_1,\varsigma_{0m}) \qquad \text{(P243.34)}$$

where $\varsigma_{0m} = z_{0m} / L_S$ and $\varsigma_1 = (z_1 + z_{0m})/L_S$ (P243.35m)

similarly (P243.32'-33') can be integrated from the height where the surface temperature is defined, i.e. $z' = z_{0h} - z_{0m}$ to the bottom model level at $z' = z_1$ (or equivalently (P243.32-33)

from $z = z_{0h}$ to $z = z_{0m} + z_1$) to obtain

$$\frac{k\left(T_{L1} - T_* + (g/c_P)\,(z_1 - z_{0h} + z_{0m})\right)}{T_S}$$

$$= \int_{z_{0h}-z_{0m}}^{z_1} \frac{\phi_h\left((z' + z_{0m}) / L_S\right)}{(z' + z_{0m})} dz'$$

$$= \int_{\varsigma_{0h}}^{\varsigma_1} \frac{\phi_h(\varsigma')}{\varsigma'} d\varsigma' \equiv (\varsigma_1,\varsigma_{0h}) \qquad \text{(P243.36)}$$

and $$\frac{k\,(q_W - q_*)}{q_S} = \Phi_h\,(\varsigma_1,\varsigma_{0h}) \qquad \text{(P243.37)}$$

where $\varsigma_{0h} = z_{0h} / L_S$ (P243.35h)

Substituting for v_S , T_S and q_S in (P243.34,36 and 37), expressions for c_D and c_H are obtained in terms of the functions ϕ_m and Φ_h :

$$c_D = \left(k/\Phi_m\,(\varsigma_1,\varsigma_{0m})\right)^2 \qquad \text{(P243.38)}$$

$$c_H = \left(k/\Phi_m(\varsigma_1,\varsigma_{0m})\right) \left(k/\Phi_h(\varsigma_1,\varsigma_{0h})\right) \qquad \text{(P243.39)}$$

As $B_S \rightarrow 0$, neutral conditions are approached and $\varsigma \rightarrow 0$ for all non-zero, finite z. It is known that the similarity functions, ϕ_X , approach unity in this limit and so, in neutral conditions,

$$c_D = c_{DN} = \left(k/\ln\,(z_1 + z_{0m}) / z_{0m} \right)^2 \qquad \text{(P243.40)}$$

$$c_H = c_{HN} = \left(k/\ln\left((z_1 + z_{0m})/z_{0m}\right)\right) \left(k/\ln\left((z_1 + z_{0m})/z_{0h}\right)\right) \qquad \text{(P243.41)}$$

The quantity ς_1 is a non-dimensional measure of the stability of the surface layer. However, it is not convenient for use in a surface layer parametrization in a numerical model since it is defined in terms of the surface fluxes which are the quantities which need to be calculated. Using (P243.35m), (P243.29), (P243.28) and (P243.8), ς_1 can be written as

$$\varsigma_1 = \frac{kc_H}{c_D^{3/2}} Ri_B \qquad \text{(P243.42)}$$

where the bulk Richardson number of the surface layer, Ri_B , is defined in terms of the surface layer buoyancy difference (given by (P243.25)) and wind shear by

$$Ri_B = (z_1 + z_{0m})\ \Delta B \ / \left| v_1 - v_0 \right|^2 \qquad \text{(P243.43)}$$

(Note that in the definition of ΔB (see (P243.25)), $z_B = z_1 - z_{0h} + z_{0m}$.) Unlike ζ_1 , the bulk Richardson number is a suitable measure of the surface layer stability for an atmospheric model since it is can be calculated readily from the basic model variables.

There still remains the need to specify c_D and c_H in terms of their respective neutral values and the Richardson number. In principle this can be done by deriving the functions Φ_m and Φ_h from the empirically determined similarity functions. However, evaluating Φ_m and Φ_h given Ri_B , z_1 , z_{0m} and z_{0h} would have to be done iteratively since (P243.42) relating Ri_B to ζ_1 involves the bulk transfer coefficients. The iteration could be done outside the model proper to generate a look-up table for c_D and c_H . This is what was done in previous versions of the UK Meteorological Office's climate and NWP models. This approach has now been abandoned in favour of specifying c_D and c_H directly as functions of Ri_B , z_1 , z_{0m} and z_{0h} . This gives greater flexibility for adjusting the stability dependence and is also more efficient computationally. It is also easy to build into the functions appropriate asymptotic behaviour at the extremes of stability.

For stable conditions with Ri_B greater than a critical value, theoretical arguments imply that turbulence should not exist. However, these arguments are strictly for homogeneous and steady states. The size of gridboxes of NWP and climate models allows considerable sub-gridscale inhomogeneity in the stability of the surface layer and in other surface parameters, particularly over land. The work of Mahrt (1987) suggests that inhomogeneity leads to some turbulence being present even for very stable surface layers. The dependence of c_D and c_H on stability as Ri_B becomes large and positive has therefore been chosen to be a decreasing function which only tends to zero for infinite Ri_B . With this specification, the surface never completely becomes turbulently decoupled from the atmosphere.

As $\left| v_1 - v_0 \right|$ in unstable conditions "free convection" takes place. In terms of the Richardson number the free convective limit corresponds to $Ri_B \rightarrow -\infty$. In this limit the friction velocity is no longer an appropriate scaling parameter in formulae relating surface layer gradients to surface fluxes. Use is made instead of the free convective velocity scale, v_{Sf} , defined by

$$v_{Sf} = \left(zF_{B^*} / \rho_*\right)^{1/3} = \left((z' + z_{0m})\ F_{B^*} / \rho_*\right)^{1/3}$$

(P243.44)

where F_{B^*} is the buoyancy flux given by (P243.23). Free convective scalings for temperature, moisture and buoyancy can be defined by analogy with (P243.13), (P243.14) and (P243.26):

$$T_{Sf} = -H_* / (\rho_* v_{Sf}\ c_P) \quad \text{(P243.45)}$$

$$q_{Sf} = -E_* / (\rho_* v_{Sf}) \quad \text{(P243.46)}$$

$$B_{Sf} = -F_* / (\rho_* v_{Sf}) \quad \text{(P243.47)}$$

By dimensional analysis the gradients of temperature and moisture variables in the free convective limit can be written as

$$\frac{\partial T_L}{\partial z'} + \frac{g}{c_P} = \frac{hT_{Sf}}{(z' + z_{0m})}$$

(P243.48)

$$\frac{\partial q_W}{\partial z'} = \frac{hq_{Sf}}{(z' + z_{0m})} \quad \text{(P243.49)}$$

where h is a dimensionless empirical constant. (P243.48-49) can be regarded as free-convective analogues to (P243.32'-33'). Integrating from $z' = z_{0f} - z_{0m}$ to the bottom model layer at $z' = z_1$ gives H_* and E_* in the form of (P243.13) and (P243.14) with c_H given by

$$c_H = f(z_1, z_{0m}, z_{0f})(-Ri_B)^{1/2} \quad \text{(P243.50)}$$

where

$$f(z, z_{0m}, z_{0f}) = \left\{3h\left[\left(\frac{z + z_{0m}}{z_{0f}}\right)^{1/3} - 1\right]\right\}^{3/2} \quad \text{(P243.51)}$$

For $z + z_{0m} \gg z_{0f}$

$$f(z, z_{0m}, z_{0f}) \simeq (3h)^{-3/2}(z_{0f} / (z + z_{0m}))^{1/2} \quad \text{(P243.52)}$$

z_{0f} is a free convective roughness length; for land and sea-ice it is assumed equal to z_{0h} .

Currently the model sets a constant value of 1.3x10m for z_{0f} over sea. As can be seen from (P243.50) and (P243.52) this gives

$$c_H \left|v_1 - v_0\right| \sim const.x(-\Delta B)^{1/2} \text{ as } \left|v_1 - v_0\right| \to 0 \quad \text{(P243.501)}$$

This form of dependence on the buoyancy difference in free convective conditions over sea is not thought to be correct. In low wind conditions heat and moisture transfers from the smooth sea surface are bounded below by molecular diffusivities. Partly motivated by dimensional arguments, the following hypotheses are made for the free convective molecular limits for the surface transfer coefficients

$$c_H \sim (3h)^{-2} \left[a_m \kappa \left(\frac{-B_S}{v_S^3} \right) \right]^{1/2} \quad as \quad v_S \to 0 \qquad \text{(P243.502)}$$

and

$$c_H \sim (3h)^{-2} \left[a_m v \left(\frac{-B_S}{v_S^3} \right) \right]^{1/2} = c_H \left(\frac{a_m v}{a_h \kappa} \right)^{1/2} \quad as \quad v_S \to 0 \qquad \text{(P243.503)}$$

where κ and v are the molecular heat conductivity and viscosity of air repectively

and a_h and a_m are dimensionless constants.

Substituting for B_S and v_S using (P243.28) and (P243.8), expressions (P243.502 and 503) become

$$c_H \left| v_1 - v_0 \right| \sim 3(h)^{-4/3} \left[\left(a_h^2 \kappa^2 / (a_m v) \right) (-\Delta B) \right]^{1/3} \quad as \quad \left| v_0 - v_1 \right| \to 0 \qquad \text{(P243.504)}$$

and

$$c_D \left| v_1 - v_0 \right| \sim 3(h)^{-4/3} \left[\left(a_h \kappa \; a_m v \right) (-\Delta B) \right]^{1/3} \quad as \quad \left| v_0 - v_1 \right| \to 0 \qquad \text{(P243.505)}$$

Using (P243.24) and (P243.504) it can be shown that

$$F_{B*} \sim \rho_* \left(a_h^2 \kappa^2 / (a_m v) \right)^{1/3} \left(-\Delta B \; (3h) \right)^{4/3} \quad as \quad \left| v_0 - v_1 \right| \to 0 \qquad \text{(P243.506)}$$

which has the same dependence on ΔB as Godfrey and Beljaars (1991). (P243.504) can be obtained from (P243.50) if z_{0f} is parametrized as

$$z_{0f} = \left(a_h^2 \; \kappa^2 / (a_m v) \right)^{2/3} \left(-\Delta B / (3h) \right)^{-1/3}$$

(P243.507)

$\kappa = 1.85 \; x \; 10^{-5} \; m^2 \; s^{-1}$ and $v = 1.34 \; x \; 10^{-5} \; m^2 \; s^{-1}$; the other empirical constants are $(3h)^{3/2} = 4.0, \; a_h = 2.360 \; x \; 10^{-2}, \; a_m = 6.837 \; x \; 10^{-2}$. These values combine to give

$$z_{0f} = b_{mo1} \; (-\Delta B)^{-1/3} \qquad \text{(P243.508)}$$

where $b_{mo1} = 4.016 \; x \; 10^{-4}$. The values chosen give a free convective latent heat flux of

$38.44\ W\ m^{-2}$ if there is a virtual potential temperature lapse of $1.5\ K$ in the surface layer and a specific humidity lapse of $7\ x\ 10^{-3}\ kg\ kg^{-1}$ and a sea surface temperature of $303.16\ K$. It is planned to replace the constant sea value of z_{0f} by the parametrization (P243.508) in a future version of the model if tests are successful.

Now that the neutral values and asymptotic limits of the bulk transfer coefficients have been established, simple analytic formulae for them can be proposed. The formulae used in the model are:

$$c_D(Ri_B\ ,\ z_1\ ,\ z_{0m}\ ,\ z_{0f}) = c_{DN}\, f_m \qquad \text{(P243.53)}$$

$$c_H(Ri_B\ ,\ z_1\ ,\ z_{0m}\ ,\ z_{0h}\ ,\ z_{0f}) = c_{HN}\, f_h \qquad \text{(P243.54)}$$

where the neutral values c_{DN} and c_{HN} are given by (P243.40) and (P243.41) respectively. The stability factors f_m and f_h are given by

$$\left.\begin{aligned} f_m &= 1 \,/\, (1 + A_m\ Ri_B) \\ f_h &= 1 \,/\, (1 + A_h\ Ri_b) \end{aligned}\right\} \qquad for\ Ri_B \geq 0$$

(P243.55)

$$\left.\begin{aligned} f_m &= 1 - A_m\ Ri_B \,/\, (1 + B_m(-Ri_B)^{1/2}) \\ f_h &= 1 - A_h\ Ri_B \,/\, (1 + B_h(-Ri_B)^{1/2}) \end{aligned}\right\} \quad for\ Ri_B < 0 \qquad \text{(P243.56)}$$

where

$$B_m = D_m\ A_m\ c_{DN} \,/\, f(z_1\ ,\ z_{0m}\ ,\ z_{0f}) \qquad \text{(P243.57)}$$

$$B_h = A_h\ c_{HN} \,/\, f(z_1\ ,\ z_{0m}\ ,\ z_{0f})$$

For speed of computation the approximate form of f given by (P243.52) is used. Both f_m and f_h and their first derivatives with respect to Ri_B are continuous. A_m and A_h are empirical constants which determine these derivatives at neutrality: $f_m{}'(0) = -A_m$ and $f_h{}'(0) = -A_h$. A_m and A_h have both been set to 10 in the model. Mahrt (1987) concludes that the effect of inhomogeneities over a gridbox is to make the slope of the $f_m(Ri_B)$ and $f_h(Ri_B)$ curves less steep in near neutral conditions than observations at a single site would imply. The parameters A_m and A_h should therefore be regarded as "tunable" rather than fixed by site measurements.

In the stable limit, i.e. as $Ri_B \to \infty$, the prescription of the stability functions given by

(P243.55) ensures that the bulk transfer coefficients decrease with increasing stability but never reach zero for finite Richardson number. There is evidence that the model's downward heat and moisture fluxes in stable conditions over sea are too large. Since sub-gridscale inhomogeneities are less over sea it may be that the stability functions should should distinguish between land and sea, in particular that for $Ri_B > 0$ the functions should approach zero more rapidly over sea (McFarlane et al. (1992)).

In the free convection limit, i.e. as $Ri_B \to \infty$, the bulk transfer coefficients have the following behaviour:

$$c_D \sim D_m^{-1} \int (z_1 , z_{0m} , z_{0f}) \, (-Ri_B)^{1/2}$$

The behaviour of c_H is in agreement with (P243.50). The constant D_m has been set to 2 in the model; it has been introduced to take into account the observed fact that the drag coefficient is smaller than the transfer coefficient for heat in unstable conditions. (Note that

$D_m = (a_h \kappa / a_m \nu)^{1/2}$) ; see (P243.503).)

(v) The treatment of the surface flux of moisture

Equation (P243.36) implies that the surface flux of moisture is given by

$$E_*/\rho_* = -c_H|v_1 - v_0|(q_{W1} - q_*) \qquad \text{(P243.59)}$$

The surface specific humidity, q_* , is not easy to predict explicitly and its implied value is inextricably linked to the parametrization of surface hydrology. When the surface is sea, sea-ice or snow covered land q_* is assumed to be $q_{SAT}(T_*,p_*)$, i.e. the saturation value corresponding to the surface temperature and pressure. Also for land when $q_{W1} \succ q_{SAT}(T_*,p_*)$ the flux is negative, i.e. from the atmosphere onto the surface, and q_* is again set to the saturated value. In all these cases the factor ψ in (P243.12) is therefore equal to 1.

For land surfaces with a positive moisture flux the "resistance method" of Monteith (1965) is invoked to calculate, E_S , the turbulent flux into the atmosphere from the soil moisture store; this gives

$$\frac{E_S}{\rho_*} = \frac{(q_* - q_{W1})}{r_A} = \frac{(q_{SAT}(T_*,p_*) - q_*)}{r_S} = \frac{(q_{SAT}(T_*,p_*) - q_{W1})}{r}$$

(P243.60)

where by comparison with (P243.59) the **aerodynamic resistance**, r_A , is given by

$$r_A = (c_H|v_1 - v_0|)^{-1} \qquad \text{(P243.61)}$$

This quantity represents the efficiency of the atmospheric turbulence in the evaporation process.

It is easy to deduce from (P243.60) that $r = r_A + r_S$. The surface or **stomatal resistance** to evaporation, r_S , characterizes the physiological control of water loss through a plant community. In effect, r_S represents all the stomata of all the leaves acting in parallel so that the plant community acts like a "giant leaf". The source of moisture in the transpiration process is the sub-stomatal cavity of the leaf where the air is saturated, or nearly so, unless the plant is under severe water stress or is dessicated. Therefore, to use this method the specific humidity in the sub-stomatal cavity is assumed to be $q_{SAT}(T_*,p_*)$.

The effective value of r_S for a gridbox is in reality a complicated function of the type and condition of the vegetation, the soil moisture content, the near surface air temperature and humidity and the amount of solar radiation reaching the surface. A future version of the model will include an interactive parametrization of r_S . However, in the current version it is a climatologically prescribed, geographically varying quantity depending on the vegetation type only. The Documentation Paper No. 70 describes its derivation from the Wilson and Henderson-Sellers (1985) global archive of land cover and soils data. The model does include a dependence of the surface moisture flux on the soil moisture content but not through the surface resistance parameter. Instead a soil moisture availability factor, ψ_S , is introduced into the equation for the flux of moisture from the soil to the atmosphere:

$$E_S = \rho_* \psi_S (q_{W1} - q_{SAT}(T_*,p_*)/r_A + r_S \qquad \text{(P243.62)}$$

where

$$\psi_S = \begin{cases} 0 & 0 \le \chi \prec \chi_W \\ (\chi - \chi_W)/(\chi_C - \chi_W) \quad \text{for} & \chi_W \prec \chi \prec \chi_C \\ 1 & \chi_C \le \chi \end{cases}$$

(P243.63)

In the model the dimensionless volumetric soil moisture concentration, χ, is defined in terms of the oil moisture available for evapotranspiration, m, by

$$\chi = \begin{cases} m/(\rho_W D_R) + \chi_W & \text{if } D_R \succ 0 \\ \chi_W & \text{otherwise} \end{cases} \qquad \text{(P243.64)}$$

D_R is the root depth of vegetation and ρ_W is the density of liquid water. χ_W is the residual value of χ at the wilting point, i.e. that value of χ below which it becomes impossible for vegetation to remove moisture from the soil. χ_C is a critical value of χ below which the flux of soil moisture to the surface or the plant roots is restrained (i.e. below which $\psi_S \prec 1$). χ_W and χ_C are climatologically prescribed, geographically varying parameters depending on the soil type and D_R is a similar parameter depending on vegetation type. Documentation Paper No. 70 describes their derivation from the Wilson and Henderson-Sellers (1985) global archive of land cover and soils data.

(P243.63) can be rewritten using (P243.64) as

$$\psi_S = \begin{cases} 0 & m \preceq 0 \ \text{ or } m_C \preceq 0 \\ m/m_C & \text{for } \ 0 \prec m \prec m_C \\ 1 & m_C \preceq m \end{cases} \qquad \text{(P243.65)}$$

where m_C is the critical soil moisture content defined by

$$m_C = \rho_W D_R(\chi_C - \chi_W) \qquad \text{(P243.66)}$$

When the model has a fully interactive stomatal resistance the factor ψ_S will not be required.

Equation (P243.62) parametrizes the flux of moisture which comes from the subsurface water, i.e. the soil moisture, store. The model also represents the effect of a surface water store. This includes a vegetative canopy store as well as water lying on the soil surface directly exposed to the atmosphere. The surface water store is commonly called the "canopy" and this name will be used in this document but the reader should not be misled into assuming this refers only to a vegetated surface. The documentation for the canopy and surface hydrology component (P252) describes how rainfall is intercepted by this store. The water in the surface store evaporates with only aerodynamic resistance since this water does not go through the soil, root and leaf stomata system. The gridbox mean "canopy evaporation" is defined to be

$$\hat{E}_A = f_A E_P \qquad \text{(P243.67)}$$

where

$$f_A = \begin{cases} c/c_M & \text{if } c_M \succ 0 \\ 0 & \text{otherwise} \end{cases} \qquad \text{(P243.68)}$$

and c is the canopy water content, c_M is the "canopy capacity" (strictly the capacity of the surface water store) and E_p is the gridbox mean "potential evaporation" given by

$$E_p/\rho_* = -c_H|v_1 - v_0|\left(q_{W1} - q_{SAT}(T_*,p_*)\right) = -\left(q_{W1} - q_{SAT}(T_*,p_*)\right)/r_A \quad \text{(P243.69)}$$

The canopy capacity is a climatologically prescribed, geographically varying parameter depending on the vegetation fraction and type. Documentation Paper No. 70 describes its derivation from the Wilson and Henderson-Sellers (1985) global archive of land cover and soils data. The canopy evaporation acting over a model timestep, δt, could deplete more canopy water than exists. To reduce this possibility the definition of f_A given by (P243.68) is replaced, when $E_p \succ 0$ and $c_M \succ 0$, by

$$f_A = c/(c_M + E_p\delta t) \quad \text{(P243.68')}$$

(P243.67) can be rewritten for convenience as

$$\hat{E}_A = \rho_* c_{EA}|v_1 - v_0|\left(q_{W1} - q_{SAT}(T_*,p_*)\right) \quad \text{(P243.70)}$$

where $c_{EA} = f_A c_H$
(P243.71)

The gridbox mean flux of water from the soil is given by

$$\hat{E}_S = (1-f_A)E_S \quad \text{(P243.72)}$$

$$= \rho_* c_{ES}|v_1 - v_0|\left(q_{W1} - q_{SAT}(T_*,p_*)\right) \quad \text{(P243.73)}$$

where

$$c_{ES} = (1-f_A)f_S c_H \quad \text{(P243.74)}$$

and

$$f_S = \frac{\psi_S}{(1+r_S/r_A)} \quad \text{(P243.75)}$$

The total flux of moisture from a land gridbox is then

$$E_* = \hat{E}_A + \hat{E}_S = -\rho_* c_E|v_1 - v_0|\left(q_{W1} - q_{SAT}(T_*,p_*)\right) = -\left(q_{W1} - q_{SAT}(T_*,p_*)\right)/r_{tot} \quad \text{(P243.76)}$$

where

$$c_E = \psi c_H, \qquad r_{tot} = r_A/\psi \quad \text{(P243.77)}$$

and

$$\psi = f_A + (1-f_A)f_S \qquad (P243.78)$$

From now on the "hat" will be left off the gridbox mean fluxes. The moisture flux from the soil before weighting with the factor $1-f_A$ is required in component P245; (P243.62) can be rewritten for convenience as

$$E_{SL} = -c_{ESL}|v_1-v_0|\left(q_{W1} - q_{SAT}(T_*,p_*)\right) \qquad (P243.79)$$

where

$$c_{ESL} = f_S c_H \qquad (P243.80)$$

(P243.76)-(P243.78) can also be used to express the surface moisture flux for a general gridpoint if f_A, ψ_S and r_S are specified appropriately. For all land points ψ_S is given by (P243.65) and r_S is set according to the vegetation type. For snow-free land points where $\Delta q = q_{W1} - q_{SAT}(T_*,p_*) \prec 0$, i.e. where the moisture flux (in terms of timelevel n quantities) is positive, f_A is given by (P243.68). For snow-covered land or land where $\Delta q \geq 0$, f_A is set to1 since it is assumed that sublimation of snow and negative moisture fluxes (i.e. downwards from the atmosphere to the surface) have only aerodynamic resistance. For sea points (whether or not sea-ice is present), $f_A = 1$, $\psi_S = 0$ and $r_S = 0$ (implying $f_S = 0$ and $\psi = 1$).

From (P243.43) and (P243.25) it can be seen that the bulk Richardson number, Ri_B, involves the factor ψ and hence, unless $f_A = 1$, $\psi_S = 0$ or $r_S = 0$, c_H is involved. However, c_H cannot be calculated until the Richardson number is known. To avoid costly iteration the neutral value of the bulk transfer coefficient for heat, c_{HN}, given by (P243.41) is used in the formulae for f_S and ψ for land points *only when these are used in calculating* Ri_B. As soon as the stability dependent c_H has been calculated, f_S and ψ can be recomputed for obtaining the moisture fluxes. (Perhaps a more satisfactory way around this would be to store and use c_H from the previous timestep to calculate Ri_B.)

(v0) <u>Surface fluxes at sea-ice points</u>

At sea points where the sea-ice fraction, f_1, is greater than zero separate surface fluxes of heat and moisture are calculated for the sea-ice and leads parts of the gridbox. This done because the ice floes generally have much lower surface temperatures than the leads. This means that the contribution of the leads to the gridbox mean fluxes can be very large despite their (usually small) fractional area.

The gridbox mean surface surface temperature is stored and updated in the model. The temperature of the leads is assumed to be the freezing point of sea-water, T_{FS}, since this is the temperature at which ice and sea-water can co-exist in equilibrium. The surface temperature of the sea-ice is therefore given by

$$T_{*(I)} = \left(T_* - (1-f_1)T_{FS}\right)/f_1 \qquad (P2430.1)$$

This is a rearrangement of equation (P241.2) giving T_* as the weighted mean of the leads and sea-ice surface temperatures.

The Richardson numbers for the leads and sea-ice components are given in terms of T_{FS} and $T_{*(I)}$ respectively by

$$Ri_{B(L)} = g(z_1+z_{0m(SEA)})\left\{\tilde{\beta}_{T1}\left(T_{L1}-T_{FS}+(g/c_p)(z_1-z_{0h(SEA)}+z_{0m(SEA)}\right)\right)$$

$$+\ \tilde{\beta}_{Q1}\psi\left(q_{W1}-q_{SAT}(T_{FS},p_*)\right)\Big\}/|v_1-v_0|^2 \qquad \text{(P2430.2)}$$

$$Ri_{B(I)} = g(z_1+z_{0(SICE)})\Big\{\tilde{\beta}_{T1}\left(T_{L1}-T_{*(I)}+(g/c_p)z_1\right)$$

$$+\ \tilde{\beta}_{Q1}\psi\left(q_{W1}-q_{SAT}(T_{*(I)},p_*)\right)\Big\}/|v_1-v_0|^2 \qquad \text{(P2430.3)}$$

($\psi = 1$ in (P2430.2 and 3) since these Richardson numbers are calculated for sea points only - see the discussion following (P243.80) in section (v)).

The leads and sea-ice surface fluxes of heat, moisture and momentum are calculated using gridbox mean surface transfer coefficients $\prec c_H \succ$, and $\prec c_D \succ$.
These are linear combinations of the corresponding coefficients calculated for ice-free sea (L), typical Marginal Ice Zone (MIZ) broken sea-ice and complete sea-ice cover (I).

For $0 \prec f_I \succ 0.7$

$$\prec c_H \succ = \left(f_I c_{H(MIZ)} + (0.7-f_I)c_{H(L)}\right)/0.7 \qquad \text{(P2430.4)}$$

$$\prec c_D \succ = \left(f_I c_{D(MIZ)} + (0.7-f_I)c_{H(L)}\right)/0.7 \qquad \text{(P2430.5)}$$

and for $0.7 \le f \le 1$

$$\prec c_H \succ = \left((1-f_I)c_{H(MIZ)} + (f_I-0.7)c_{H(I)}\right)/0.3 \qquad \text{(P2430.6)}$$

$$\prec c_D \succ = \left((1-f_I)c_{D(MIZ)} + (f_I-0.7)c_{D(I)}\right)/0.3 \qquad \text{(P2430.7)}$$

where

$$c_{H(L)} = c_H\left(Ri_{B(L)},z_1,z_{0m(SEA)},z_{0h(SEA)},z_{0f(SEA)}\right) \qquad \text{(P2430.8)}$$

$$c_{H(MIZ)} = c_H\left(Ri_{B(i)},z_1,z_{0(MIZ)},z_{0(MIZ)},z_{0(MIZ)}\right) \qquad \text{(P2430.9)}$$

$$c_{H(i)} = c_H\left(Ri_{B(i)},z_1,z_{0(SICE)},z_{0(SICE)},z_{0(SICE)}\right) \qquad \text{(P2430.10)}$$

and similarly for the drag coefficients c_D . The sea roughness lengths and $z_{0(MIZ)}$ and $z_{0(SICE)}$ are specified in Appendix B. The heat and moisture fluxes for the leads are calculated using

$$H_{*(L)}/(\rho_* c_p) = -(1-f_I)\prec c_H \succ |v_1 - v_0|\Big(T_{L1} - T_{FS}$$

$$+\ (g/c_p)(z_1 - z_{0h(SEA)} + z_{0m(SEA)})\Big) \qquad \text{(P2430.11)}$$

$$E_{*(L)}/\rho_* = -(1-f_I)\psi\langle c_H\rangle|v_1 - v_0|\left(q_{W1} - q_{SAT}(T_{FS},p_*)\right)$$

(P2430.12)
and those for the ice floes using

$$H_{*(I)}/(\rho_* c_p) = -f_I\langle c_H\rangle|v_1 - v_0|\left(T_{L1} - T_{*(I)} + (g/c_p)z_1\right)$$

(P2430.13)

$$E_{*(I)}/\rho_* = -f_I\psi\langle c_H\rangle|v_1 - v_0|\left(q_{W1} - q_{SAT}(T_{*(I),p_*})\right)$$

(P2430.14)

The weights $1-f_I$ and f_I are used so that the gridbox mean contributions from the leads and the ice are obtained. The total fluxes for gridboxes with sea-ice are then

$$H_* = H_{*(L)} + H_{*(I)} \qquad \text{(P2430.15)}$$

$$E_* = E_{*(L)} + E_{*(I)} \qquad \text{(P2430.16)}$$

and the gridbox mean latent heat flux is $L_C E_* + L_F E_{*(I)}$. The surface stress is given by (P243.6) with $\langle c_D\rangle$ in place of c_D.

The use of the mean transfer coefficients to calculate the separate leads and ice fluxes is based on the assumption that patches of water and ice are small compared with the distances required for the turbulent flow to become adapted to one surface type. The linear interpolations, (P2430.4-7), to obtain the gridbox mean surface transfer coefficients are intended to represent, very simply, the dependence on sea-ice fraction shown in Figure 10 of Andreas et al. (1984).

(vi) The calculation of wind, temperature and humidity at standard heights in the surface layer

The Monin-Obukhov theory outlined above provides a basis for interpolating atmospheric variables within the surface layer in a manner consistent with the calculation of the surface turbulent surfaceturbulent fluxes. It is useful for the model to output temperature and humidity at 1.5 m above the height where $T = T_*$ and winds at 10 m above the the height where $v = v_0$ since these values can then be readily compared with observations.

Using (P243.34) and (P243.36-37) the following equations for $v(z')$, $T_L(z')$ and $q_W(z')$ are easily obtained:

$$v(z') = v_0 + c_{DR}(z')(v_1 - v_0) \tag{P243.81}$$

$$T_L(z') = T_* - (g/c_p)(z' - z_{0h} + z_{0m})$$

$$+c_{HR}(z')\left(T_{L1} - T_* + (g/c_p)(z_1 - z_{0h} + z_{0m})\right) \tag{P243.82}$$

$$q_W(z') = q_{W1} + \psi(c_{HR}(z') - 1)\left(q_{W1} - q_{SAT}(T_*,p_*)\right) \tag{P243.83}$$

where

$$c_{DR}(z') = \frac{\phi_m\left(\zeta,\zeta_{0m}\right)}{\phi_m\left(\zeta_1,\zeta_{0m}\right)} \tag{P243.84}$$

$$c_{HR}(z') = \frac{\phi_h\left(\zeta,\zeta_{0h}\right)}{\phi_h\left(\zeta_1,\zeta_{0h}\right)} \tag{P243.85}$$

$$\zeta = (z' + z_{0m})/L_S \tag{P243.86}$$

and ζ_1 , ζ_{0m} , ζ_{0h} are given by (P243.35).

By inverting (P243.38-39) the following are obtained

$$\phi_m\left(\zeta_1,\zeta_{0m}\right) = k/c_D^{1/2} \tag{P243.87}$$

$$\phi_h\left(\zeta_1,\zeta_{0h}\right) = kc_D^{1/2}/c_H \tag{P243.88}$$

The method of Geleyn (1988) is used to obtain approximate expressions for $\phi_m\left(\zeta,\zeta_{0m}\right)$ and $\phi_h\left(\zeta,\zeta_{0h}\right)$. Geleyn's analysis needs generalising so that it can treat different surface roughness lengths for momentum and heat.

To obtain the approximate expressions for ϕ_m and ϕ_h Geleyn uses

$$\phi_m = \phi_h = \begin{cases} 1+(\alpha/L_S)(z'+z_{0m}) & \text{for stable conditions}(L_S \geq 0) \\ 1/[1-(\alpha/L_S)(z'+z_{0m})] & \text{for unstable conditions}(L_S \prec 0) \end{cases}$$

so that integrals like (P243.34) and (P243.36) are easy to obtain analytically. The resulting expressions with $\zeta = \zeta_1$ are substituted into (P243.87-88) and rearranged to give an expression for α/L_S in terms of the c_D and c_H evaluated in the model. α/L_S can then be eliminated from the expression for the ratios of ϕ's in (P243.84-85). Finally the following are obtained after putting $z' = 10$ for the wind interpolation and $z' = 1.5 + z_{0h} - z_{0m}$ for the temperature and humidity interpolations:

$$c_{DR}(10m) = \frac{10}{z_1} + \frac{c_D^{1/2}}{k}\left[\ln\left(\frac{10 + z_{0m}}{z_{0m}}\right) - \frac{10}{z_1}\ln\left(\frac{z_1 + z_{0m}}{z_{0m}}\right)\right]$$

(P243.89)

$$c_{HR}(1.5m) = \frac{1.5}{z_B} + \frac{c_H}{kc_D^{1/2}}\left[\ln\left(\frac{1.5 + z_{0h}}{z_{0h}}\right) - \frac{1.5}{z_B}\ln\left(\frac{z_1 + z_{0m}}{z_{0h}}\right)\right]$$

(P243.90)

in stable conditions (i.e. when $Ri_B \geq 0$) and

$$c_{DR}(10m) = 1 - \frac{c_D^{1/2}}{k}\ln\left[\frac{10(z_1 + z_{0m}) + z_{0m}(z_1 - 10)\exp(k/c_D^{1/2})}{z_1(10 + z_{0m})}\right]$$

(P243.91)

$$c_{HR}(1.5m) = 1 - \frac{c_H}{kc_D^{1/2}}\ln\left[\frac{1.5(z_1 + z_{0m}) + z_{0h}(z_B - 1.5)\exp(k/c_D^{1/2}/c_H)}{z_B(1.5 + z_{0h})}\right]$$

(P243.92)

in unstable conditions (i.e. when $Ri_B \prec 0$).

In the expressions for c_{HR},

$$z_B = z_1 + z_{0m} - z_{0h} \qquad \text{(P243.93)}$$

At sea points with sea-ice the Ri_B , c_D and c_H which the model code calculates are those appropriate for the sea-ice part of the gridbox. When the sea-ice fraction, $f_I = 1$ this is obviously correct. When leads are present ($0 \prec f_I \prec 1$) the interpolation formulae still use the sea-ice values of c_D , c_H , z_{0m} and z_{0h} since to calculate the gridbox mean values would involve a lot of extra calculations merely to obtain a slightly better approximation to a purely diagnostic quantity.

(vii) The calculation of the wind mixing energy flux at sea points

The wind mixing energy flux is the rate of production of turbulent kinetic energy per unit area in the sea surface layer by the wind stress at the air-sea interface. In atmosphere only configurations this quantity is a useful diagnostic from the model. When the atmospheric model is coupled to an ocean model the wind mixing energy flux has to be accumulated over an ocean model timestep and then used in the calculation of the mixing in the upper layers of the ocean. The gridbox mean wind mixing energy flux, F_{WME}, is given by

$$F_{WME} = (1 - f_I)\underline{\tau}*\upsilon_S \qquad \text{(P243.94)}$$

where f_I is the fraction of the gridbox covered in sea-ice, $\underline{\tau}*$ is the surface stress defined by (P243.6) and υ_S is the sea surface friction velocity, defined in the same way as that for the atmosphere (see (P243.7)):

$$\upsilon_S = (\underline{\tau}*/\rho_{SEA})/|\underline{\tau}*/\rho_{SEA}|^{1/2} \qquad \text{(P243.95)}$$

ρ_{SEA} is the density of sea water, taken to be $1000\ \mathrm{kg\ m^{-3}}$. It has been assumed that the surface stress is continuous across the air-sea interface.

(P243.94) and (P243.95) can be combined to obtain

$$F_{WME} = (1-f_I)|\underline{\tau}*|^{3/2}/\rho_{SEA}^{1/2} \quad \text{(P243.96)}$$

The magnitude of the surface stress can be seen from (P243.6) to be given by

$$|\underline{\tau}*| = \rho_* c_D |v_1 - v_0|^2 \quad \text{(P243.97)}$$

(P243.96) is evaluated using timelevel n variables and the velocities in (P243.97) are those interpolated to the p-grid.

At sea-ice points the magnitude of the surface stress, $|\underline{\tau}*|$ given by (P243.97) and used in (P243.96) as well as the corresponding v_S^2 (= $|\underline{\tau}*|/\rho_*$) given by (P243.B7) and used in the Charnock formula (P243.B6) are both calculated using the drag coefficient for the leads fraction of the gridbox, $c_{D(L)}$, rather than the gridbox mean value, $\prec c_D \succ$.

(viii) <u>Boundary layer fluxes and mixing coefficients</u>

Setting X in (P243.2) to the vector horizontal wind, $\mathbf{v}$, the liquid/frozen water static energy, $s_L = c_P T_L + gz$, and the total water content, q_W, in turn gives

$$\underline{\tau}/\rho = -\mathbf{F}_v/\rho = -\overline{w'v'} = K_M \frac{\partial v}{\partial z} \quad \text{(P243.98)}$$

$$F_{TL}/\rho = \overline{w'T'_L} = -K_H \left(\frac{\partial T_L}{\partial z} + \frac{g}{c_p} \right) \quad \text{(P243.99)}$$

$$F_{qw}/\rho = \overline{w'q_W'} = -K_H \frac{\partial q_W}{\partial z} \quad \text{(P243.100)}$$

for the local turbulent mixing fluxes at model layer interfaces 1+1/2 to BL_LEVELS - 1/2. The fluxes are assumed to be zero for the interfaces k+1/2 when k $\geq$ BL_LEVELS. Formulating the turbulence mixing using the cloud-conserved thermodynamic and water content variables automatically includes the effects of cloud water phase changes on the turbulence (Yamada and Mellor, 1979).

The atmospheric densities at layer interfaces are required for calculating the fluxes using (P243.98)-(P243.100). $\rho_{k-1/2}$ for 2 $\leq$ k $\leq$ BL_LEVELS are calculated using

$$\rho_{k-1/2} = p_{k-1/2}/(RT_{Vk-1/2}) \quad \text{(P243.116)}$$

where

$$p_{k-1/2} = A_{k-1/2} + B_{k-1/2} p_*$$

and the virtual temperature at the layer interface is found by linear interpolation in z

$$T_{Vk-1/2} = \left(T_{Vk-1}(z_k - z_{k-1/2}) + T_{Vk}(z_{k-1/2} - z_{k-1}) \right) / (z_k - z_{k-1})$$

$$= (T_{Vk-1}\Delta z_{k-1/4} + T_{Vk}\Delta z_{k-3/4})/\Delta z_{k-1/2} \quad \text{(P243.117)}$$

Appendix A gives the layer thicknesses and half-layer thicknesses.

The turbulent mixing coefficients in (P243.98-100) are given by

$$K_M = \mathcal{L}_M^2 f_M(Ri)\left|\frac{\partial v}{\partial z}\right| \qquad (P243.101)$$

$$K_H = \mathcal{L}_M \mathcal{L}_H f_H(Ri)\left|\frac{\partial v}{\partial z}\right| \qquad (P243.102)$$

The stability functions f_H and f_M are empirically specified in terms of the Richardson number,Ri, and given by

$$f_H = f_M = 1/(1+G_0 Ri) \qquad Ri \geq 0 \qquad (P243.103)$$

$$\left.\begin{array}{l} f_H = 1 - G_0 Ri/(1+(G_0/E_H)(\mathcal{L}_M/\mathcal{L}_H)(-Ri)^{1/2}) \\ f_M = 1 - G_0 Ri/(1+(G_0/E_M)(\mathcal{L}_M/\mathcal{L}_H)(-Ri)^{1/2}) \end{array}\right\} \qquad Ri < 0$$

(P243.104)

f_H and f_M and their first derivatives are continuous for all values of Ri. G_0 , E_H and E_M are adjustable parameters currently set to 10, 25 and 4 respectively. In the very stable limit $(Ri \to \infty)$ the functions become small but never reduce to zero and cut off all mixing. In the opposite very unstable limit $(Ri \to -\infty)$ the wind shear drops out of the expressions for K_M and K_H,

$$K_M \sim E_M \mathcal{L}_H \mathcal{L}_M (-Ri)^{1/2} |\partial v/\partial z| = E_M \mathcal{L}_M \mathcal{L}_H)(-\partial B/\partial z)^{1/2}$$

(P243.105)

and

$$K_H \sim E_H \mathcal{L}_H^2 (-Ri)^{1/2} |\partial v/\partial z| = E_H \mathcal{L}_H^2)(-\partial B/\partial z)^{1/2} \qquad (P243.106)$$

where the Richardson number, Ri, and buoyancy gradient, $\partial B/\partial z$, across the layer interface are defined below.

f_H is set to 1, i.e. the mixing coefficient, K_M, is set to its neutral value if the convective cloud amount is greater than zero and the model layer interface is within or at the base of the convective cloud. This is to ensure that mixing of momentum is allowed in regions of convective cloud. The current convection scheme does not mix momentum. Also because the latent heating in convective cloud leads to a temperature profile which the boundary layer turbulence scheme sees as stable, momentum mixing would be reduced to small values at and above convective cloud base if the stability dependence of f_M were not overridden.

$\mathcal{L}_M$ and $\mathcal{L}_H$ are neutral mixing lengths given by the Blackadar formula

$$\mathcal{L}_M(z') = \frac{k(z' + z_{0m})}{1 + k(z' + z_{0m})/\lambda_M} \qquad (P243.107)$$

$$\mathcal{L}_H(z') = \frac{k(z' + z_{0m})}{1 + k(z' + z_{0m})/\lambda_H} \qquad (P243.108)$$

where the von Karman constant, $k = 0.4$ and λ_M and λ_H are asymptotic neutral mixing lengths proportional to the boundary layer depth, z_h, except for shallow boundary layers:

$$\lambda_{H,M} = \max(40.0, 0.15 z_H) \qquad \text{(P243.109)}$$

The roughness length in (P243.107) and (P243.108) are given in Appendix B. (Note that for sea points with sea-ice the roughness lengths are set to the sea-ice values even if sea-ice fraction is less than one, i.e. leads are present.)

Near the surface simple finite difference calculations for the vertical gradients can become inaccurate because of the quasi-logarithmic profiles of variables (Arya (1991)). This can be seen by writing generalised forms of (P243.31'-33') above the surface layer as

$$\frac{\partial X}{\partial z'} = \frac{A}{\mathcal{L}_X} = \frac{A}{k(z' + z_{0m})} + \frac{A}{\lambda_X} \qquad \text{(P243.110)}$$

This can be integrated between heights z_{k-1} and z_k (assuming A is can be regarded as a constant within this range of z') to obtain

$$\Delta X_{k-1/2} = A\left[\frac{1}{k}\ln\left(\frac{z_k + z_{0m}}{z_{k-1} + z_{0m}}\right) + \frac{\Delta z_{k-1/2}}{\lambda_X}\right] \qquad \text{(P243.111)}$$

Substituting for A in (P243.110) using (P243.111) and evaluating the gradient at $z' = z_{k-1/2}$ results in

$$\left.\frac{\partial X}{\partial z'}\right|_{k-1/2} = \left.\frac{\tilde{\mathcal{L}}_{X,k-1/2}}{\mathcal{L}_{X,k-1/2}}\frac{\Delta X}{\Delta z'}\right|_{k-1/2} \qquad \text{(P243.112)}$$

$$\tilde{\mathcal{L}}_{X,k-1/2} = \frac{k\Delta z_{k-1/2}}{\left[\ln\left(\frac{z_k + z_{0m}}{z_{k-1} + z_{0m}}\right) + \frac{k\Delta z_{k-1/2}}{\lambda_X}\right]} \qquad \text{(P243.113)}$$

where

To obtain accurate values for the fluxes, F_X, the gradients given by (P243.112) should be used, so from (P243.98-100) and (P243.101-102)

$$F_X = -\tilde{\mathcal{L}}_X \tilde{\mathcal{L}}_M f_X(\mathrm{Ri})\left|\frac{\Delta v}{\Delta z}\right|\frac{\Delta X}{\Delta z} \qquad \text{(P243.114)}$$

So when finite vertical differences are used in (P243.101-102) the mixing lengths $\tilde{\mathcal{L}}_X$ should give more accurate fluxes than $\mathcal{L}_X$. The modified mixing lengths given by (P243.113) are only used for $2 \le k \le$ K_LOG_LAYR, i.e. for the model layer interfaces K_LOG_LAYR $- 1/2$ and below. K_LOG_LAYR is an integer parameter of the model (K_LOG_LAYR $\le$ BL_LEVELS).

The Richardson number is defined by

$$\mathbf{Ri} = (\partial B/\partial z)/|\partial \mathbf{v}/\partial z|^2$$

$$\frac{\partial B}{\partial z} \equiv g\left(\tilde{\beta}_T\left(\frac{\partial T_L}{\partial z} + \frac{g}{c_p}\right) + \tilde{\beta}_Q\frac{\partial q_W}{\partial z}\right) \qquad \text{(P243.115)}$$

where
The buoyancy parameters $\tilde{\beta}_T$ and $\tilde{\beta}_Q$ are given by (P243.18)-(P243.21). Note that the stability depends on gradients of the cloud-conserved variables and, when there is cloud present $(C \succ 0)$, allows for latent heating and cloud water loading effects through the buoyancy parameter β_C. In the special case of a cloud-free region the modified Richardson number becomes the more familiar "dry" Richardson number and the turbulence scheme reduces to the more usual formulation.

In the finite difference formulation the expression for the Richardson number, (P243.110), needs the values of the buoyancy parameters $\tilde{\beta}_T$ and $\tilde{\beta}_Q$ at layer interfaces. The way these are calculated from the values in the layers on either side of the interface is important, particularly when one layer is cloudy and the other clear. The finite difference form of (P243.110) and its implications for the scheme's ability to represent cloud top entrainment instability are discussed in Appendix C.

The boundary layer top is set at the first model layer interface (above the surface) where $\mathbf{Ri} \succ \mathbf{Ri}_C$. ($\mathbf{Ri}_C$ is set to 1.) As a safeguard, the top is set at the interface $\mathrm{BL\_LEVELS} - 1/2$ if the Richardson number has not exceeded the critical value below this. The boundary layer depth, z_h, (used in P243.109) is set to the height above the surface of the layer interface so diagnosed.

In Scheme 2 (selected by setting *IF definition A03_2C) an integer variable, N_{rml}, is set to the number of model layers beneath the boundary layer top. If the surface buoyancy flux (F_{B*}, given by (P243.23)) is zero or negative this variable is set to zero. If $N_{rml} \geq 1$ tests are done to see if the heat and moisture fluxes at the surface and at the boundary layer top imply a deepening of the boundary layer in the timestep. The tests are based on a modified Richardson number across the previously diagnosed boundary layer top (at level $N_{rml} + 1/2$) which is calculated as

$$\mathit{Ri}_{k-1/2} = \mathrm{Ri}^n_{k-1/2} + \frac{g\Delta z_{k-1/2}}{|\Delta \mathbf{v}^n_{k-1/2}|}\Big\{\tilde{\beta}_{T,k-1/2}\Big[(\delta T_L,k)_{ex} - (\delta T_{L,k-1})_{ex}\Big]$$

$$+ \tilde{\beta}_{Q,k-1/2}\Big[(\delta q_{W,k})_{ex} - (\delta q_{W,k-1})_{ex}\Big]\Big\} \qquad \text{(P243.200)}$$

where $k = N_{rml} + 1$.

This calculation is only done if $N_{rml} \leq \mathrm{BL\_LEVELS} - 2$ since if $N_{rml} = \mathrm{BL\_LEVELS} - 1$ the model does not allow a deeper boundary layer so no deepening tests should be done. The "explicit" increments in (P243.200) are given by

$$(\delta X_k)_{ex} = (g\delta t/\Delta p_k)(F^n_{X,k+1/2} - F^n_{X,k-1/2}) \qquad \text{(P243.201)}$$

$$(\delta X_{k-1})_{ex} = (g\delta t/\Delta p_{rml})(F^n_{X,k-1/2} - F^n_{X*}) \qquad \text{(P243.202)}$$

again with $k = N_{rml} + 1$;
$X = T_L$ or q_W and

$$\Delta p_{rml} = \sum_{k=1}^{N_{rml}} \Delta p_k \qquad \text{(P243.203)}$$

The superscript n denotes a flux calculated in terms of timelevel n variables. (N.B. T_L and q_W are not updated with the explicit increments.)

If $Ri_{k-1/2} \le Ri_C$, z_h is reset to $z_h + \Delta z_k$ and N_{rml} is incremented by 1, i.e. the boundary layer is deepened by one model layer. Also if $Ri_{k-1/2} \le Ri_C$ and $Ri_{k-1/2} \le Ri_{k-1/2}$, $Ri_{k-1/2}$ is reset to $Ri_{k-1/2}$ so that local mixing between layers $k - 1$ and k is not inhibited by using a large Richardson number. The turbulent mixing coefficients and fluxes are recalculated using the reset Richardson number in (P243.103-104).

This process is repeated to see whether the boundary layer can deepen further (each iteration only deepens the boundary layer by at most one model layer). At most $BL\_LEVELS - 2$ iterations are necessary since $1 \le N_{rml} \le BL\_LEVELS - 1$.

(ix) Implementation in the code.

Firstly *BDY_LAYR* calls subroutine Z to calculate the layer thicknesses, Δz_k , the lower-half-layer thicknesses, $\Delta z_{k-1/4}$, using (P243.A2,3,7), and the heights of the layer interfaces above the surface, $\Delta z_{k+1/2}$, using (P243.A4), all for $k = 1$ *to BL_LEVELS* . The reciprocal level separations, $1 / \Delta z_{k-1/2}$, for $k = 2$ *to BL_LEVELS* are calculated in *BDY_LAYR* using (P243.A6) and also the height of the first model level above the surface, z_1 , using (P243.A9). These quantities are p-grid values since they are all derived from the hydrostatic equation which relates z to the p-grid quantities Π and Θ_V . Time level n values of $\Pi, T, q, q_C^{(L)}$ and $q_C^{(F)}$ are used, i.e. the values on entry to *BDY_LAYR* .

Then subroutine *UV_TO_P* is called to interpolate the wind components u_k, v_k for levels $k = 1$ *to BL_LEVELS* to the p-grid. The uv-grid values are not overwritten; a "hat" (^) indicates an interpolated value on the p-grid. Subroutines *SICE_HTF* and *SOIL_HTF* (components P241 and P242 respectively) are then called to calculate the subsurface heat fluxes for sea-ice and land points.

Subroutine *SF_EXCH* then calculates the surface exchange coefficients and the "explicit" surface fluxes of momentum, heat and moisture (all in terms of timelevel n variables). The values of z_{0m}, z_{0h} and z_{0f} are set as described in Appendix B. The liquid/frozen water temperature, T_L , and total water content, q_W , for model layer 1 are then calculated using the definitions (P243.9) and (P243.10) followed by α_L, a_L, β_T, β_Q, β_C, $\hat{\beta}_T$ and $\hat{\beta}_Q$ for model layer 1 using (P243.18)-(P243.21).

Subroutine *UV_TO_P* is called from *SF_EXCH* to interpolate the surface velocity components, u_0 and v_0 , to the p-grid. The temperature lapse across the surface layer, ΔT , the water content lapse, Δq , and the magnitude of the surface layer wind shear, v_{sh} , are then calculated as follows:

$$\Delta T = T^n_{L1} - T^n_{*} + g(z^n_1 + z_{0m} - z_{0h})/c_P$$

(P243.118) $$\Delta q = q^n_{W1} - q_{SAT}(T^n_{*}, p^n_{*})$$

(P243.119) $$v_{sh} = \max(10^{-3}, \left| \hat{v}^n_1 - \hat{v}^n_0 \right|)$$

(P243.120) Note that v_{sh} is calculated on the p-grid; the minimum value of 10^{-3} m s^{-1} is a safeguard to ensure that the Richardson number (which has the shear squared in the denominator - see (P243.43)) does not become very large or infinite. At sea points with sea-ice fraction greater than zero the timelevel n sea-ice surface temperature is calculated using (P2430.1) and then separate lapses for the leads and sea-ice parts of the gridbox are calculated as follows:

for the leads

$$\Delta T_{(LEADS)} = T^n_{L1} - T_{FS} + g(z^n_1 + z_{0m(SEA)} - z_{0h(SEA)})/c_P$$

(P2430.17) $$\Delta q_{(LEADS)} = q^n_{W1} - q_{SAT}(T_{FS}, p^n_{*})$$

(P2430.18) and for the ice

$$\Delta T = T^n_{L1} - T^n_{*(I)}\ g z^n_1/c_P \qquad \text{(P2430.19)}$$

$$\Delta q = q^n_{W1} - q_{SAT}(T^n_{*(I)}, p^n_{*})$$

(P2430.20) (Note that the lapses for the ice at sea-ice points are put in the same stores as the lapses calculated for non-sea-ice points.)

The factors f_A, ψ_S, f_S and ψ defined in section (v) are calculated next. As explained in section (v), at this stage f_S and ψ have to be computed using the neutral surface exchange coefficient for heat, c_{HN} , given by (P243.41), for land points with positive moisture flux. The bulk Richardson number for the surface layer given by (P243.43) is calculated using the definition of the surface layer buoyancy difference given by (P243.25). Note that at sea-ice points it is the Richardson number for the sea-ice part of the gridbox which is calculated here.

This Richardson number, the previously set z_{0m}, z_{0h}, z_{0f} and z_1 are input to subroutine $FCDCH$ to find the surface exchange coefficients c_D and c_H . $FCDCH$ calculates c_D and c_H using (P243.40,41,52,53,54,55,56,57). f_S is recalculated using c_H rather than c_{HN} and f_A is recalculated using (P243.68')for use in subsequent calculations.

The coefficients, c_{HR} and c_{DR} , needed for interpolating temperature, humidity and the wind in the surface layer are calculated by calling subroutine SFL_INT under the control of logical variables to indicate whether these diagnostics are required for the current timestep. SFL_INT uses (P243.89-92). $c_{DR}(10m)$ is interpolated to the uv-grid using subroutine P_TO_UV . (Again note that for sea-ice points the interpolation coefficients strictly only apply to the sea-ice part of the gridbox but, as explained at the end of section (vi), they are used for calculating the standard height surface layer diagnostics for sea-ice points even when the sea-ice fraction is less than one.)

If the sea-ice fraction is greater than zero at a sea point the Richardson number for the leads part of the gridbox is calculated using (P2430.2) and then $c_{H(L)}$, $c_{H(MIZ)}$, $c_{D(L)}$, $c_{D(MIZ)}$ as prescribed by (P2430.8) and (P2430.9) and their equivalents for the drag coefficients. The gridbox mean surface transfer coefficients, $<c_H>$ and $<c_D>$ are calculated using (P2430.4-7)

Next the atmospheric density at the surface, $\rho_* = p_*^n/(RT_*^n)$, is calculated and then the quantities:

$$RK_M(1) = \rho_* c_D v_{sh}$$

(P243.124) $$RK_H(1) = \rho_* c_H v_{sh}$$

(P243.125) $$RK_{EA} = \rho_* c_{EA} v_{sh} = f_A RK_H(1)$$

(P243.126) $$RK_{ESL} = \rho_* c_{ESL} v_{sh} = f_S RK_H(1)$$

(P243.127) $RK_{ES} = \rho_* c_{ES} v_{sh} = (1 - \int_A) RK_{ESL}$

(P243.128) $RK_E = \rho_* c_E v_{sh} = RK_{EA} + RK_{ES}$

(P243.129) (For sea-ice points with leads the surface transfer coefficients are the gridbox mean values). Use is made of (P243.71), (P243.74), (P243.80) and (P243.77) respectively to obtain the second forms of (P243.126-129). v_{sh} is given by (P243.120).

For sea points the wind mixing energy flux and the sea surface momentum roughness length are calculated next. $\left| \underline{\tau}_* \right|$ is needed for both these quantities and is calculated using the following form of (P243.97)

$$\left| \underline{\tau}_* \right| = RK_M(1) v_{sh}$$

(P243.130) The wind mixing energy flux, F_{WME}, is then obtained using (P243.96); note that F_{WME} is a p-grid quantity. $\left| v_s \right|^2$ is calculated for use in the formula for z_{0m} (see Appendix B) from

$$\left| v_s \right|^2 = \left| \underline{\tau}_* \right| / \rho_* \qquad \text{(P243.131)}$$

$z_{0m(SEA)}$ is set using (P243.B6) and passed out of subroutine BDY_LAYR to be used in the following timestep for calculating c_D and c_H for sea points.

$RK_M(1)$ is then interpolated onto the uv-grid by calling P_TO_UV ; the interpolated quantity is denoted by an overbar. The "explicit" surface fluxes are calculated using timelevel n quantities:

$$\tau_{x*}^n \equiv \tau_x^n(1) = \overline{RK_M(1)}(u_1^n - u_0^n)$$

(P243.132) $\tau_{y*}^n \equiv \tau_y^n(1) = \overline{RK_M(1)}(v_1^n - v_0^n)$

(P243.133) $H_*^n / c_P \equiv F_{TL}^n = -RK_H(1) \Delta T$

(P243.134) $E_*^n \equiv F_{qW}^n(1) = -RK_E \, \Delta q$

(P243.135) $E_A^n = -RK_{EA} \, \Delta q$

(P243.136) $E_S^n = -RK_{ES} \, \Delta q$

(P243.137) $E^{n}_{SL} = -RK_{ESL}\ \Delta q$

(P243.138)

where ΔT and Δq are given by (P243.118) and (P243.119) respectively.

The area weighted "explicit" surface heat and moisture fluxes for the sea-ice and leads are calculated using, for the sea-ice:

$$H^{n}_{*(I)}\ /c_{P} \equiv F^{n}_{TL(I)}(1) = -f_{I}\ RK_{H}(1)\Delta T \qquad \text{(P2430.21)}$$

$$E^{n}_{*(I)} \equiv F^{n}_{qW(I)}(1) = -f_{I}\ RK_{E}(1)\Delta q$$

(P2430.22) where ΔT and Δq are given by (P2430.19) and (P2430.20), and for the leads:

$$H^{n}_{*(L)}\ /c_{P} \equiv F^{n}_{TL(LEADS)}(1) = -(1-f_{I})RK_{H}(1)\Delta T_{(LEADS)}$$

(P2430.23) $E^{n}_{*(L)} \equiv F^{n}_{qW(LEADS)}(1) = -(1-f_{I})RK_{E}\ \Delta q_{(LEADS)}$

(P2430.24) where $\Delta T_{(LEADS)}$ and $\Delta q_{(LEADS)}$ are given by (P2430.17) and (P2430.18).

The total "explicit" surface heat and moisture fluxes at sea-ice points are found by adding the weighted sea-ice and leads components. These calculations conclude subroutine SF_EXCH .

Subroutine $KMKH$ then calculates the turbulent mixing coefficients and the "explicit fluxes" of momentum, heat and moisture in the boundary layer. As with the surface layer calculations in SF_EXCH , these are all time-level n quantities. Firstly in a loop over levels $k = 2\ to\ BL\_LEVELS$ the buoyancy parameters $\tilde{\beta}_{Tk}$ and $\tilde{\beta}_{Qk}$ defined by (P243.18) - (P243.21) are calculated.

As described in Appendix C, these "full-level" parameters need to be interpolated to layer interfaces (or "half-levels") using (P243.C3). In a second loop over levels $k = 2\ to\ BL\_LEVELS$ the weights W_{k-1} and W_{k} defined by (P243.C5) are calculated and then the layer interface buoyancy parameters $\tilde{\beta}_{Tk-1/2}$ and $\tilde{\beta}_{Qk-1/2}$. ($\tilde{\beta}_{T1}$ and $\tilde{\beta}_{Q1}$,needed for the level 1+1/2 parameters, are input from SF_EXCH .) The Richardson number, $Ri_{k-1/2}$ is calculated using (P243.C1) and (P243.C2). If the difference in winds (on the p-grid), $\left|\ \Delta\hat{v}\ \right|_{k-1/2}$, is found to be less than 10^{-6} m s^{-1} it is set to this value; this is to ensure that a very small number or zero is not used in thedenominator of the expression for the Richardson number. If the buoyancy difference, $\Delta B_{k-1/2}$, turns out to be positive the weights are both reset to 0.5 and the buoyancy difference recalculated before calculation of the Richardson number; the reason for this is explained in Appendix C. The Richardson numbers are immediately used in a test to set the boundary layer depth. Before this second loop is entered a logical variable $TOPBL$ is set to false. For each pass through the loop, if $TOPBL$ = .false. and $Ri_{k-1/2}$ is greater than 1 or k is equal to BL_LEVELS then $TOPBL$ is set to true and the boundary layer depth, z_h , is set to $z_{k-1/2}$, i.e. to the height above the surface of the top of layer k .

The calculations described in this paragraph are done by calling subroutine EX_COEF if Scheme 2 is selected. In Scheme 1 they are done with in-line code. The asymptotic mixing lengths, λ_M and λ_H , are calculated in terms of z_h according to (P243.109). In a loop over levels $k = 2 \ to \ BL\_LEVELS$ the mixing coefficients ρK_M and ρK_H are calculated for half levels $k - 1/2$. The density, $\rho_{k-1/2}$, is calculated according to (P243.116,117). The neutral mixing lengths for half levels $k - 1/2$ are calculated from (P243.107,108) and (P243.113).. The Richardson numbers $Ri_{k-1/2}$ are used to calculate the stability functions f_M and f_H as given by (P243.103) and (P243.104) for each half level. $f_{M,k-1/2}$ is set to 1 if there is convective cloud in layer k . Finally the stability dependent turbulent mixing coefficients are calculated:

$$RK_M(k) = \left(\rho \tilde{\mathfrak{L}}_M^2 f_M | \Delta \hat{v} | / \Delta z \right)_{k-1/2}$$

(P243.141)

$$RK_H(k) = \left(\rho \tilde{\mathfrak{L}}_H \tilde{\mathfrak{L}}_M f_H | \Delta \hat{v} | / \Delta z \right)_{k-1/2}$$

(P243.142) where the reciprocal level separations $1/\Delta z_{k-1/2}$ are input from BDY_LAYR . The coefficients with index k are those for half level $k - 1/2$ for $k = 2 \ to \ BL\_LEVELS$.

After the turbulent mixing coefficients have been obtained the "explicit" local fluxes of heat and moisture are calculated in a loop over levels $k = 2 \ to \ BL\_LEVELS$:

$$F^n_{TL}(k) = -RK_H(k) \left((T^n_{Lk} - T_{Lk-1}) / \Delta z_{k-1/2} + g/c_P \right)$$

(P243.145) $$F^n_{qW}(k) = -RK_H(k)(q^n_{Wk} - q_{Wk-1}) / \Delta z_{k-1/2}$$

(P243.146) The fluxes with index k are those across layer interface $k - 1/2$, i.e. from layer $k - 1$ to layer k. In Scheme 2 only, tests are then done to see whether the boundary layer can deepen in the timestep. In an iteration loop from $1 \ to \ BL\_LEVELS - 2$ the "explicit" T_L and q_W increments are calculated if $1 \le N_{rml} \le BL\_LEVELS - 2$ for levels $k = N_{rml}$ and $N_{rml} + 1$ using (P243.201-203). These are used to calculate the modified Richardson number for the interface between layers N_{rml} and $N_{rml} + 1$ given by (P243.200). (The original Richardson number is not overwritten at this stage.) z_h and N_{rml} are incremented if the modified Richardson number is less than or equal to the critical value and the original Richardson number reset to the modified value if the the latter is less, as explained in Section P243(viii). Resetting these variables concludes the iteration loop for deepening tests in Scheme 2. The Richardson number at the final boundary layer top is then modified in Scheme 2 by multiplying it by $\min(100.0/\Delta z_{k-1/2} \ , \ 1)$ as a simple way of adjusting for insufficient vertical resolution to represent the stability of inversions - see Appendix C.

In Scheme 2 there is a second call to subroutine EX_COEF to recalculate the turbulent mixing coefficients since the values of the Richardson number and boundary layer height may have been changed as a result of the deepening tests. The "explicit" fluxes of heat and moisture are recalculated with the reset mixing coefficients using (P243.145,146).

The quantities $RK_M(k)$ and $\Delta z_{k-1/2}$ (for $k = 2 \ to \ BL\_LEVELS$) are interpolated to the uv-grid for calculating the "explicit" fluxes of momentum in the boundary layer. An overbar in what follows indicates such an interpolated quantity. The "explicit" fluxes are finally calculated in a further loop over levels $k = 2 \ to \ BL\_LEVELS$:

$$\tau_x^n(k) = \overline{RK_M(k)}(u_k^n - u_{k-1}^n) \ / \ \overline{\Delta z_{k-1/2}}$$

(P243.143) $$\tau_y^n(k) = \overline{RK_M(k)}(v_k^n - v_{k-1}^n) \ / \ \overline{\Delta z_{k-1/2}}$$

(P243.144) The fluxes with index k are those across layer interface $k - 1/2$, i.e. from layer $k - 1$ to layer k . These calculations conclude subroutines $KMKH$.

Appendix A. The calculation of the layer thicknesses, the layer interface heights and level separations.

The hydrostatic equation written in terms of the Exner pressure function, $\Pi = (p/100000)^{\kappa}$, is

$$\frac{\partial \Pi}{\partial z} = -\frac{g}{c_p \Theta_V} \qquad \text{(P243.A1)}$$

where Θ_V is the **virtual potential temperature** defined by

$$\Theta_V = (T/\Pi)\left(1 + (\eta^{-1} - 1)q - q_C^{(L)} - q_C^{(F)}\right) \qquad \text{(P243.A2)}$$

So in finite differences the thickness of layer k, Δz_k, is calculated using

$$\Delta z_k = -(c_p/g)\Theta_{Vk}\Delta\Pi_k \qquad \text{(P243.A3)}$$

where Θ_{Vk} is Θ_V at level k and $\Delta\Pi_k = \Pi_{k+1/2} - \Pi_{k-1/2}$.

The heights of the layer interfaces above the surface are given by

$$z_{k+1/2} = z_{k-1/2} + \Delta z_k \qquad \text{(P243.A4)}$$

with $z_{1/2} = 0$ representing the surface.

The distance between levels $k-1$ and k (for $k \geq 2$) is defined by

$$\Delta z_{k-1/2} = (z_k - z_{k-1}) = (z_k - z_{k-1/2}) + (z_{k-1/2} - z_{k-1}) \qquad \text{(P243.A5)}$$

$$= \Delta z_{k-1/4} + (\Delta z_{k-1} - \Delta z_{k-5/4}) \qquad \text{(P243.A6)}$$

The form (P243.A6) is convenient for computation with the lower half-thicknesses, $\Delta z_{k-1/4}$, given by

$$\Delta z_{k-1/4} = (z_k - z_{k-1/2}) = -(c_p/G)\Theta_{Vk}(\Pi_k - \Pi_{k-1/2}) \qquad \text{(P243.A7)}$$

where the Exner pressure at level k is given by

$$\Pi_k = \frac{\Delta(\Pi p)_k}{(k+1)\Delta p_k} = \frac{\Pi_{k+1/2}p_{k+1/2} - \Pi_{k-1/2}p_{k-1/2}}{(k+1)(p_{k+1/2} - p_{k-1/2})} \qquad \text{(P243.A8)}$$

which is consistent with the geopotential equation (see Documentation Paper No.10, Eqn.26).

z_1 , the height of the first atmospheric level above the surface, is obtained by using (P243.A7) with $k = 1$:

$$z_1 = \Delta z_{3/4} = (c_p/g)\Theta_{V1}(\Pi_1 - \Pi_*) \quad \text{(P243.A9)}$$

Appendix B. The specification of the roughness lengths.

To evaluate the surface exchange coefficients c_D and c_H roughness lengths for momentum, z_{0m} , heat and moisture, z_{0h} , and free convective turbulence, z_{0f} are required.

For **land points** the roughness lengths are set to

$$z_{0m} = z_0$$

$$z_{0h} = z_0$$

(P243.B1)

$$z_{0f} = z_{0h}$$

(except in the mesoscale version of the model - see below) where

$$z_0 = \begin{cases} \max(\min(z_{0V}, 5x10^{-4}), z_{0V} - 4x10^{-4} S) & if\ S < 5x10^3 \\ Z_{0V} & otherwise \end{cases}$$

(P243.B2)

S is the the mass of snow per unit area in kg m^{-2}, and z_{0V} is the roughness length representing the effects of vegetation and very small-scale surface irregularities (not orography). z_{0V} is a climatologically prescribed, geographically varying quantity depending on the vegetation and land use. Documentation Paper No.70 describes its derivation from the Wilson and Henderson-Sellers (1985) global archive of land cover and soils data.

Note that the model currently sets equal values for the heat/moisture roughness length, z_{0h} , and that for momentum, z_{0m} at land points (except in the mesoscale version of the model - see below). It has been established empirically that the "vegetative" value of z_{0h} is much smaller, by up to an order of magnitude, than z_{0V} . This is because the form drag effects of small-scale irregularities and obstacles (hedges, isolated trees, buildings etc.) are included in the empirically determined z_{0V} . The specification of surface roughness lengths is a topic of current research; it is expected to introduce effective roughness for land points representing all unresolved surface features in a future version of the model.

The dependence of the roughness on the snow areal density represents very crudely the surface smoothing effect of snow. Eagleson (1970) gives values for the roughness length of 5x10^{-5} m and10^{-3} m for smooth snow on short grass and snow on prairie respectively. Kuz'min (1972) suggests that for stable snow cover, 0.1 - 0.2m thick, $z_0 = 5x10^{-4}\ m$ increasing to 2.5x10^{-3} m and 6x10^{-3} m where snow is patchy and where grass protrudes respectively. Clearly the roughness depends on the depth of snow relative to the height of the vegetation, with complete cover yielding a value between 5×10^{-5} m and 5×10^{-4} m. The former is probably an extreme value for a completely smooth snow cover so the higher value would be a more generally appropriate lower limit, except where the snow free value is lower. Thompson et al.(1981) suggest that as a rough approximation $z_0 \simeq 0.1h$ where h is the typical vegetation height. Although inspection of data given by Eagleson (1970) shows that there is considerable variability in the relationship between vegetation roughness and height the above relationship can be used as a reasonable first approximation.

If z_{0S} is the modified roughness in the presence of snow of actual depth h_S then using Thompson's approximation:

$$z_{0S} = 0.1(h - h_S) \text{ with } z_{0S(MIN)} = \min(0.1h,\ 5x10^{-1}) \qquad \text{(P243.B3)}$$

$0.1h$ is identified with the roughness due to vegetation, z_{0V} . In the model the actual depth of snow is not stored, so h_S is set to S/ρ_{SNOW} where ρ_{SNOW} is the density snow. Eagleson (1970) gives ρ_{SNOW} = 200 to 280 kg m^{-3} during the period December to January, and Geiger (1966) gives average values varying from 200 to 350 kg m^{-3} at snowmelt. New snow generally has a density between 100 and 200 kg m^{-3} . A uniform value of 250 kg m^{-3} is assumed so that equation (P243.B3) reduces to (P243.B2).

For points classified as land-ice in the Wilson and Henderson-Sellers (1985) archive the areal density of snow is initialised to 5x10 kg m^{-3} . This is done to ensure that such points never become snow or ice free. At permanent land-ice points the land surface parameters, in particular the surface roughness, are already set to values appropriate for the snow and ice covered surface and so no further reduction in the roughness should be made. Therefore z_0 is set to z_{0V} if the areal density of snow, S, is greater than 5x10 kg m^{-3}.

The specification given by (P243.B1-2) does not represent the form drag effects of unresolved orography through effective values for the roughness lengths. It is intended that a future version of the model will do this in a theoretically consistent way. As an interim measure, the mesoscale version of the model (and only this version) uses orographic momentum roughnesses for Britain as derived by Smith and Carson (1977) in place of z_{0V} . Other land points on the mesoscale model grid have z_{0m} set to 0.1 m. In this version of the model z_{0m} is limited to a maximum value of 1.0 m and z_{0h} is set to $0.2z_{0m}$ with an upper limit of 0.1 m.

For **sea points with sea-ice** the three roughness lengths are set to

$$z_{0m} = z_{0(SICE)}$$

$$z_{0h} = z_{0(SICE)} \qquad \text{(P243.B4)}$$

$$z_{0f} = z_{0h}$$

Overland (1985) quotes values of the neutral drag coefficient at 10 m for various types of sea-ice in his Tableÿ6. These can be translated into roughness lengths using (P243.40) with z_1 =10 m.Overland's summary suggests that for the seasonal ice zone in the Bering Sea and the winter Arctic $c_{DN}(10m) \simeq 2.5-3.0x10^{-3}$ which gives $z_{0(SICE)} \simeq 3.4-6.7x10^{-3}m$. However, a lower value for $c_{DN}(10m)$ of 1.7x10^{-3} (corresponding to $z_{0(SICE)} = 6x10^{-4}m$) for Arctic pack ice is quoted. As a compromise the model uses $z_{0(SICE)} = 3x10^{-3}m$ (which corresponds to $c_{DN}(10m) = 2.4x10^{-3}$).

As discussed in section (v0), an additional roughness length, $z_{0(MIZ)}$, is used in the calculation of the surface transfer coefficients at sea-ice points. $z_{0(MIZ)}$ is set to 10^{-1}m.

For **sea points without sea-ice**

$$z_{0m} = z_{0m(SEA)}$$

$$z_{0h} = z_{0h(SEA)} = 10^{-4}\ m \qquad \text{(P243.B5)}$$

$$z_{0f} = z_{0f(SEA)} = 1.3x10^{-3}\ m$$

where $z_{0m(SEA)}$ is given by the formula proposed by Charnockÿ(1955), except for low wind speeds:

$$z_{0m(SEA)} = \max(z_{0h(SEA)},\ M_{Ch}v^2/g) \qquad \text{(P243.B6)}$$

v_S is the magnitude of the surface friction velocity which is defined by (P243.7) and M_{Ch} is a dimensionless constant (sometimes called the "Charnock coefficient") set to 0.012. This value is close to that recommended by Smith (1988) for non-coastal areas of sea. A generalisation of the Charnock formula which applies in all conditions including low-wind, smooth-sea conditions is (see e.g. Smith, 1988)

$$z_{0m(SEA)} = (M_{Ch}/g)v_S^2 + a_{mol}/v_S \qquad \text{(P243.B6')}$$

where a_{mol} is a constant proportional to the molecular viscosity of air, ν .

Setting $a_{mol} = 1.1.1x10^{-5}\ m\ s^{-1}$ gives a minimum value for $z_{0m(SEA)}$ of 10^{-4} m if

$M_{Ch} = 0.012$. It is planned to implement this generalisation of the Charnock formula along with the parametrization of z_{0f} involving the molecular diffusivity and viscosity of air, as given by (P243.508), in a future version of the model. It has theoretical advantages over the rather arbitrary low wind speed formulation given by (P243.B6).

v_S can only be evaluated when c_D is known but c_D depends on z_{0m} . To avoid costly iteration the value of $z_{0m(SEA)}$ given by (P243.B6) is stored, as an ancillary field, for calculating c_D and c_H using (P243.53)-(P243.57) in the following timestep. Thus for sea points v_S^2 is calculated using timelevel n quantities as

$$v_S^2 = c_D(Ri_B^n, z_1, z_{0m}, z_{0h}, z_{0f}) \left| v_1^n - v_0^n \right|^2 \qquad \text{(P243.B7)}$$

and z_{0m}^{n+1} is found by using this in (P243.B6). The velocities in (P243.B7) are those interpolated to the p-grid. At sea-ice points the drag coefficient for the leads, $c_{D(L)}$, is used rather than the gridbox mean value, $<c_D>$.

Appendix C. The finite difference expression for the Richardson number.

The finite difference form of the Richardson number defined in (P243.110) can be written:

$$Ri_{k-1/2} = \Delta z_{k-1/2} \Delta B_{k-1/2} / |\Delta v|^2 \quad for \quad 2 \le k \le BL\_LEVELS \qquad \text{(P243.C1)}$$

where the buoyancy difference, $\Delta B_{k-1/2}$, is given by

$$\Delta B_{k-1/2} = g(\tilde{B}_{Tk-1/2}(\Delta T_L + (g/c_P)/\Delta z_{k-1/2}) + \tilde{B}_{Qk-1/2} \Delta q_W) \qquad \text{(P243.C2)}$$

In (P243.C1) and (P243.C2):

$$\tilde{B}_{Tk-1/2} = W_{k-1} \tilde{B}_{Tk-1} + W_k \tilde{B}_{Tk}$$

(P243.C3)

$$\tilde{B}_{Qk-1/2} = W_{k-1} \tilde{B}_{Qk-1} + W_k \tilde{B}_{Qk}$$

$\Delta X = X_k - X_{k-1}$ for $x = v, T_L$ and q_W; $\Delta z_{k-1/2}$ is the mid-layer separation,

i.e. $z_k = z_{k-1}$, and W_k and W_{k-1} $(= 1 - W_k)$ are weighting factors for the layers. Values of the weights can be derived from the energetic stability analysis as described by Mason (1985) and as used by MacVean and Mason (1990) in an analysis of cloud-top entrainment instability through small-scale mixing. A small mass of air ϵ is exchanged between the layers and mixed. The resulting changes in potential and kinetic energies of the two layers per unit mass exchanged are calculated. The Richardson number can be defined in terms of these energy changes as

$$Ri = \frac{\delta PE/\epsilon_M}{\delta KE/\epsilon_M} \qquad \text{(P243.C4)}$$

where δ represents the change due to the mixing of mass ϵ_M and PE and KE are the potential and kinetic energies respectively. For small ϵ_M this quantity drops out of the expression for Ri and (P243.C1) is obtained with

$$W_k = \Delta z_{k-1/4} / \Delta z_{k-1/2} \quad \text{and} \quad W_{k-1} = \Delta z_{k-3/4} / \Delta z_{k-1/2} \qquad \text{(P243.C5)}$$

$\Delta z_{k-1/4}$ and $\Delta z_{k-3/4}$ are half-layer thicknesses above and below the interface between layers $k-1$ and k respectively (see Appendix A).

The Richardson number given by (P243.C1) implies instability if

$$\Delta h \le k_M L \Delta q_W \qquad \text{(P243.C6)}$$

where $h = c_{PT} + Lq + gz$ is the moist static energy and

$$k_M = 1 - \frac{c_P}{L}\frac{\tilde{B}_{Qk-1/2}}{\tilde{B}_{Tk-1/2}} \qquad \text{(P243.C7)}$$

Substituting for the buoyancy parameters $\tilde{B}$ from (P243.18)-(P243.21) we obtain

$$k_M = \frac{\left(N_1 - N_2(c_P T/L)\right)(1+L\alpha/c_P)}{\left(D_1 + D_2 \alpha T/\epsilon\right)} \qquad \text{(P243.C8)}$$

where

$$N_1 = W_{k-1}(1 - C_{k-1}) + W_k(1 - C_k),$$

$$N_2 = W_{k-1}((1 - C_{k-1})/\epsilon - 1) + W_k((1 - C_k)/\epsilon - 1)),$$

$$D_1 = W_{k-1}(1 + (L\alpha/c_P)(1 - C_{k-1})) + W_k(1 + (L\alpha/c_P)(1 - C_k)),$$

$$D_2 = W_{k-1}C_{k-1} + W_k C_k,$$

$\alpha = \partial q_{SAT}/\partial T,$ T is the mean of T_{k-1} and T_k

and C_{k-1} and C_k are cloud fractions.

Putting $C_{k-1} = C_k = 1$ in (P243.C8), $k_M = k_W$ where

$$k_W = \frac{(c_P T/L)(1+L\alpha/c_P)}{(1+\alpha T/\epsilon)} \qquad \text{(P243.C9)}$$

In this case the weights drop out of the expression for k_M and the instability criterion (P243.C6) is that derived by Randall (1980) and Deardorff (1980) for cloud top entrainment instability (CTEI). However, MacVean and Mason (1990) argue that the appropriate criterion for CTEI is that which is obtained by putting C_{k-1} and $C_k = 0$ giving

$$k_M = \frac{\left(c_P T/L + (W_k/W_{k-1})(1 - (c_P T/L)(\epsilon^{-1}-1))\right)(1 + L\alpha/c_P)}{\left((1+W_k/W_{k-1}) + ((W_k/W_{k-1})L/c_P\, T + 1/\epsilon)\alpha T\right)} \qquad \text{(P243.C10)}$$

MacVean and Mason also argue that "a self-sustaining dynamical instability is most likely when the two layers involved in the mixing process are of comparable depth". This implies that the weights should be equal for a correct calculation of stability across cloud top. Large-scale models generally have layer thicknesses increasing with height in the boundary layer so that there is a greater weighting for the upper layer (i.e. $W_k < W_{k-1}$). This biasses the stability parameter k_M given by (P243.C10) towards its dry mixing value $k_D = 1 - (c_P T/L)(\epsilon^{-1} - 1)$, obtained by putting $C_{k-1} = C_k = 0$ in (P243.C8). This can be seen by noting that the dry value is also obtained in the limit $W_k/W_{k-1} \to \infty$ in (P243.C10). (It is also worth noting that k_W is obtained in the limit $W_k/W_{k-1} \to 0$ in (P243.C10)).

For $T = 288\ k,\ k_D = 0.93,\ k_W = 0.23$ and, for $W_{k-1} = W_k$, (P243.C10) gives $k_M = 0.70$. Since Δq_W is negative for an inversion, using weights given by (P243.C5) with $\Delta z_k > \Delta z_{k-1}$ would tend to inhibit CTEI. For this reason the weights W_k and W_{k-1} are both set to 0.5 if a preliminary calculation of the Richardson number using weights given by (P243.C5) gives a positive value (indicating stability).

Inversions at the top of the boundary layer are much thinner than large- and meso-scale models can resolve. The Δz on the r.h.s. of (P243.C1) may therefore be an inappropriate length scale for the Richardson number characterising the strength of an inversion. Mason (1986) suggests "replacing the Δz with a fixed plausible length scale such as 100m." In scheme 2 (selected by setting A03_2C) this is done if $\Delta z > 200m$.

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