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GRAVITY WAVE DRAG

VERSION 2

by

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Model Version 4.1

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Modification Record		
Document version	**Author**	**Description**
unnumbered	C.A.Wilson R. Swinbank	Original (1A) scheme representing linear hydrostatic waves only.
2	S. Webster	New (3A) scheme based on Shutts(1990). Includes anisotropic orography, non-linear low level wave-breaking effects and trapped lee waves.

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1. INTRODUCTION

In October 1994, a new gravity wave drag (GWD) parametrization, together with an orographic roughness scheme (Smith and Jackson, 1996), was implemented into the Unified Model (UM) at the release of version 3.4. The new GWD scheme is based on the theoretical ideas of Shutts (1990) and supercedes the original UM GWD scheme which was based on Palmer *et al.* (1986). More recently, Milton and Wilson (1996) and Gregory *et al.* (1996) have also outlined the theory behind the new scheme whilst documenting the performance of the new parametrizations in the UM in operational forecasting mode and climate simulations respectively.

By design, the new GWD scheme is more realistic than the original scheme and thus it includes three major additional features. Firstly, the anisotropy of the subgrid-scale orography is now accounted for in the calculation of the surface stress. Secondly, a representation of low level, non-linear wave breaking effects which occur for low Froude number[1] flows is included, the parametrization being based on the hydraulic jump response seen in various modelling studies (*e.g.* Peltier and Clark, 1979). Thirdly, an additional, non-hydrostatic gravity wave stress is diagnosed which represents the effect of trapped lee waves on the large scale flow.

This document details the new GWD code as released at version 4.1. Since that code release, a couple of additional changes to the vertical deposition of stress have been implemented in the operational version of the scheme. These shall be highlighted at the appropriate point.

2. SCHEME OVERVIEW

The basic elements of the scheme are the determination of a 'surface' stress and the distribution of this stress through the atmospheric column. Both these elements have been enhanced in the new scheme. The details of these enhancements shall be briefly described below before a more in depth review is given.

The hydrostatic surface stress is now dependent on the degree of anisotropy of the subgrid-scale orography and also on the low level Froude number. Additionally, the calculation of the hydrostatic surface stress disregards that air which is perceived to be blocked by the subgrid orography, i.e. we are only concerned with that air which is perceived to go over the mountains rather than around them. A further feature of the scheme is the calculation of a non-hydrostatic surface stress associated with the initiation of trapped lee waves.

Consistent with these features, the vertical distribution of the atmospheric stress has also been significantly modified. In particular, for low Froude number flows the stress profile is based on a hydraulic jump model. Thus, the stress is reduced linearly with height up to a diagnosed hydraulic jump height. Above this height, and for points where the Froude number is large, a saturation hypothesis similar to that employed in the original scheme is invoked. The trapped lee wave stress is also reduced linearly with height, this time the height to which the reduction occurs is proportional

[1] Here the Froude number is defined as F_r=U/Nh, where U is the low level wind speed, N the buoyancy frequency and h a height scale of the subgrid-scale orography.

to the height of the trapped lee wave amplitude maximum.

3. THE DETERMINATION OF THE GRAVITY WAVE SURFACE LAYER

The first task performed by the GWD scheme is to determine the depth of the surface layer. Here, the surface layer is defined as the layer of air that is lifted over the subgrid-scale orography and is thus capable of generating gravity waves.

The top of this layer is assumed to be related to the height of the subgrid-scale orography. Thus,

$$\hat{h} = \min(\sqrt{2}\sigma, 750m) \qquad (1)$$

where $\hat{h}$ is the top of the surface layer and σ is the standard deviation of the subgrid-scale orography. The factor of $\sqrt{2}$ is included so that the top of the surface layer is close to the level of the tops of the mountains (although Wallace *et al.* (1983) suggest that 2σ may be a more appropriate value). The 750m limit is imposed so that in regions which have both large subgrid-scale orographic variations and large mean elevations, the surface layer does not reach jet stream levels[2].

The bottom of the surface layer is determined as the top of a blocked layer (if one exists), i.e.

$$h_b = \hat{h} - 0.985\frac{U}{N} \qquad (2)$$

the top of the layer of air going *around* the mountains. This height is calculated using the theory of Rottman and Smith (1989) as

where h_b is the depth of the blocked layer and U and N are the low level wind speed and Brunt Vaisala frequency.

In the code, the surface layer is determined in terms of full model levels. Thus, the top of the surface layer is taken as the full level whose *upper* layer boundary is higher than $\hat{h}$. The depth of each model level is calculated as

$$\Delta h_k = (\Delta\Pi)_k \theta_k C_p / g \qquad (3)$$

where $\Delta\Pi$ is the difference in Exner function $(\Pi = (p/p_o)^{\kappa})$ across the model level, θ is the potential temperature, C_p is the specific heat of dry air and g the acceleration due to gravity.

The bottom of the surface layer is taken as the full level whose *lower* layer boundary is higher

[2] Quite whether this is the most appropriate solution for these regions is debatable. Even though the winds may be rather strong, most of the air will probably go around the mountains.

than h_b . h_b is evaluated using the wind[3] and Brunt Vaisala frequency at level 2. The finite difference form used for the static stability, N^2 (square of the Brunt Vaisala frequency), is

$$N_k^2 = \frac{\Delta\Pi_{k-\frac{1}{2},k} N^2_{k+\frac{1}{2}} + \Delta\Pi_{k,k+\frac{1}{2}} N^2_{k-\frac{1}{2}}}{\Delta\Pi_{k-\frac{1}{2},k+\frac{1}{2}}} \quad (4)$$

$$N^2_{k+\frac{1}{2}} = \frac{g(\Theta_{k+1} - \Theta_k)}{\Theta_{k+1}\Theta_k \; \Delta\Pi_{k,k+1} \; \frac{C_p}{g}} \quad (5)$$

with

Here, the subscript notation refers to the levels over which the terms are calculated. Notice that these definitions of $\hat{h}$ and h_b restrict the surface layer to levels 2, 3 and 4 for the standard 19 level model. A further constraint in the code on the choice of the surface layer levels is that the top of the surface layer must be at least level START_L (currently level 3). As defined in the code, the surface layer is therefore much more constrained than Shutts (1990) may have intended.

Once the surface layer is known, the average density (ρ ,where $\rho_k = p_k / (R\theta_k \Pi_k)$), wind speed and buoyancy frequency of the layer are calculated. This averaging is done with respect to pressure.

4. CALCULATION OF THE HYDROSTATIC SURFACE STRESS

In this GWD scheme, the components of the hydrostatic surface stress are given by

$$\tau_{xs} = \frac{\rho_s U_0^3}{\hat{K}\sigma^2 N_s} \hat{h}_1 \hat{h}_2 \; (\sigma_{xx}\cos\chi + \sigma_{xy}\sin\chi) \quad (6)$$

$$\tau_{ys} = \frac{\rho_s U_0^3}{\hat{K}\sigma^2 N_s} \hat{h}_1 \hat{h}_2 \; (\sigma_{xy}\cos\chi + \sigma_{yy}\sin\chi) \quad . \quad (7)$$

Here, ρ_s and N_s are the surface values of the density and Brunt Vaisala frequency. U_0 is the

[3] The full wind speed is used in this calculation. Perhaps the more appropriate measure is that component of the wind perpendicular to the major axis of the orography.

component of the wind in the direction of the surface stress[4]. $\hat{k}$ is a typical wavelength for the spectrum of gravity waves being parametrized, χ is the direction of the surface layer wind relative to westerly, σ is the standard deviation of the subgrid-scale orography and σ_{xx} , σ_{xy} and σ_{yy} are the squared gradients of the elevation of the subgrid-scale orography. Thus, for example, $\sigma_{xx} = (\delta h/\delta x)^2$ where h is height of the subgrid-scale orography. $\delta h/\delta x$ is calculated at every point of the subgrid-scale orography dataset and then σ_{xx} is simply the grid-box mean of the square of $\delta h/\delta x$. The methods for calculating these fields are described in Jones (1995) and Robinson (1994). A discussion of the calculation of $\hat{k}$ will be given below.

The values of $\hat{h}_1$ and $\hat{h}_2$ depend on the low level Froude number and are calculated as

$$\hat{h}_1 = \begin{cases} \dfrac{N\sigma}{\alpha U_0} & if<1 \\ 1 & otherwise \end{cases} \tag{8}$$

$$\hat{h}_2 = \begin{cases} \dfrac{N\sigma}{\beta U_0} & if<1 \\ 1 & otherwise \end{cases} \tag{9}$$

where α and β are constants to be specified. These constants are very important because they determine which regime is invoked for a given Froude number. Currently, α is set to 0.4 and β to 1. Thus, for $F_r > 1/\alpha$ (here $F_r = U_o/N\sigma$, i.e. there is a factor of $\sqrt{2}$ difference relative to equation 1), the linear hydrostatic regime is invoked and $\tau_s \propto U_0 N_s$. Thus, the surface stress is the same as that in Palmer *et al.* (1986) apart from the inclusion of the representation of anisotropy. For $F_r \le 1/\alpha$, non-linear processes become increasingly important. The scheme is then based on the hydraulic jump response such as that seen in the downslope windstorm model of Smith (1985). For $1/\beta \le F_r \le 1/\alpha$, $\tau_s \propto U_0^2$, whilst for $F_r \le 1/\beta$ flow blocking is assumed to accompany the hydraulic jump response and $\tau_s \propto U_0^3/N_s$. Note that this is the only way in the code that the representation of flow blocking modifies the hydraulic jump response. With equations (6) and (7) so defined, the hydrostatic surface stress is a smoothly varying function of the Froude number.

We now return to the calculation of $\hat{k}$. Shutts (1990) defines $\hat{k}$ as

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[4]Shutts (1990) indicates that this should in fact be the full wind speed.

$$\kappa=\frac{1}{3}\left[\frac{k_u^{3/2}-k_l^{3/2}}{k_u^{1/2}-k_l^{1/2}}\right] \qquad \textbf{(10)}$$

where k_u is the highest wavenumber lying outside the range of trapped lee waves (see section 6) and k_l is the lowest unresolved wavenumber of the model. Shutts suggests that a wavelength of 25km might be appropriate in determining $k_u(=2\pi/25000m)$. For the current operational model $(0.83^\circ x\ 1.25^\circ)$, the longest unresolved wavelength is approximately 200km in the zonal direction in mid-latitudes, whilst at climate resolution $(2.5^\circ x\ 3.75^\circ)$ it is three times this value. Thus, taking $k_l=2\pi/200km$, then $\kappa^{-1}=8000m$ at operational resolution whilst taking $k_l=2\pi/600km$ yields $\kappa^{-1}=9500m$ at climate resolution. However, κ is ultimately a tuneable parameter for the GWD scheme. Currently at operational resolution $\kappa^{-1}=10000m$, whilst at climate resolution $\kappa^{-1}=20000m$.

5. DETERMINATION OF THE HYDROSTATIC DRAG PROFILE.

Once the surface stress is known, its deposition in the vertical can be determined. As hinted at in section 2, the vertical stress profile is dependent on the regime invoked, and that in turn depends most crucially on the constant, α . The vertical stress gradient implies a drag on the atmosphere via the following relation:-

$$\frac{\delta u}{\delta t}=g\frac{\delta\tau}{\delta p} \qquad \textbf{(11)}$$

5.1 Hydraulic Jump Regime

For $F_r\leq 1/\alpha$, the vertical profile of the atmospheric stress is based on the hydraulic jump response. Essentially, this involves diagnosing the top of the hydraulic jump layer and then linearly reducing the stress through this layer down to a prescribed fraction of the surface value. Above the hydraulic jump layer, the stress is passed onto the saturation hypothesis part of the scheme (see section 5.2).

Following Rottman and Smith (1989), the height of the hydraulic jump layer, H_0, is taken to be 3/4 of the mean vertical wavelength of the waves. Note that Rottman and Smith's theory is based on constant wind speed and buoyancy frequency, which will clearly only approximately hold in the real atmosphere. H_0 is defined as being the level when

$$\int_{h_b}^{H_0} \frac{N(z)}{U_0(z)} dz \geq \frac{3\pi}{2} \quad \textbf{(12)}$$

is first satisfied, where h_b is the depth of the blocked layer as defined in section 3.

In the code, equation (12) is evaluated as a summation of $(N \Delta z)/U_0$, where this term is calculated separately at each level. The first level included in the summation is the bottom level of the surface layer. N and Δz are calculated as in the surface layer calculations of sections 3 and 4. If no blocked layer has been diagnosed, then the depth, Δz, of level 2 additionally includes the depth of the bottom model level.

There are a number of problems of an algorithmic nature involved in this calculation in the current scheme. These are associated with the stress deposition occurring via the linear hydrostatic response, and not the hydraulic jump response, if certain criteria are met. Obviously, this then implies that the deposition of the stress is inconsistent with the surface stress calculation.

The criteria in question are:-

(i) the bottom boundary of the surface layer is unstable $(N^2 < 0)$. This should not be an issue; what is important is that the surface layer as a whole (in a vertically averaged sense) is stable. This must be true for a non-zero surface stress to have been calculated in the first place.

(ii) the current level is unstable. This check is made for all levels which, of course, includes the surface layer levels and so we have the same problem as in (i).

(iii) the jump height is diagnosed as being lower than or at level START_L. It would seem incorrect to check that equation (12) is satisfied below the top of the surface layer.

(iv) when a jump height is not found by level K_TROP (working upwards from level 2). K_TROP is defined as the first level whose upper boundary is above 250hPa, where the surface pressure is taken to be 1000hPa. It would seem more appropriate, in such a situation, to set the jump height to occur at level K_TROP. Indeed, this alternative approach was adopted as one of the changes implemented into the operational forecast model on 6/11/96.

After the jump height has been found, a check is made for the prescence of critical levels below the jump height. This calculation diagnoses whether the wind has turned through $\pi/2$ relative to the direction of the surface stress, i.e. whether $u \cdot \tau_s < 0$, where u is the wind at the current level. If one is found, then the jump height is lowered to that level.

Once the hydraulic jump height has been determined, the vertical profile of the atmospheric stress can be prescribed. For those points where a critical level was not found, 2/3 of the surface stress is deposited below the jump height. For those points where a critical level was found, all the atmospheric stress is deposited below the jump height.

It should be noted that there is little guidance available in setting the factor of 2/3. Therefore, a second change made in the operational model on 6/11/96 was to increase this factor to 5/6. This change was made in light of excessive GWD being imparted in the model stratosphere, most notably over the Himalayas in northern winter.

Shutts (1990) proposes that the stress profile below the jump height should decrease linearly from the bottom of the surface layer up to the jump height. This is **almost** what is done in the code.

There, the vertical stress gradient, $\delta\tau/\delta p$, is evaluated with δp being the depth from the bottom of level START_L to the top of the jump height level. The stress gradient in the levels below and including level START_L is then modified to include the depth of those levels below level START_L, i.e. for these levels, $\delta\tau$ is unchanged whilst δp is increased to include the depth of those levels below level START_L. This results in less stress being deposited across the levels below and including START_L than should be. Correspondingly, across those levels above START_L and below the jump height, an equal amount of excess stress is deposited.

Above the jump height, at those points which did not have a critical level below the jump height, the remaining stress is deposited via the 'saturation hypothesis' mechanism. This is the mechanism by which all the hydrostatic stress is deposited for linear hydrostatic points. The details of the saturation hypothesis are the subject of the next section.

5.2 Linear Hydrostatic Regime

For those points with $F_r > 1/\alpha$, and for hydraulic jump points above their jump height, the determination of the vertical profile of the atmospheric stress is akin to the Palmer *et al.* (1986) saturation mechanism. At each layer boundary, the maximum stress that could be supported by the mean flow is calculated. This 'critical stress' is then used to regulate the stress passed upwards through the model column. If the stress at the current layer boundary exceeds the critical stress, then it is set equal to the critical stress. Otherwise, the stress is unchanged. In other words, the stress is maintained at marginal stability as it is passed upwards through the atmospheric column.

The components of the critical stress, τ_c , at the current layer boundary are calculated as

$$\tau_{xc} = \frac{\rho U_0^3}{\kappa\sigma^2 N}(\sigma_{xx}\cos\chi + \sigma_{xy}\sin\chi)\left(\frac{\alpha}{\beta}\right) \qquad \textbf{(13)}$$

$$\tau_{yc} = \frac{\rho U_0^3}{\kappa\sigma^2 N}(\sigma_{xy}\cos\chi + \sigma_{yy}\sin\chi)\left(\frac{\alpha}{\beta}\right) \qquad \textbf{(14)}$$

where the terms as as defined in equations (6) and (7), except that U_0 , ρ and N are the wind speed in the direction of the surface stress, the density and the buoyancy frequency for the current level. Note that the factor $\hat{h}_1\hat{h}_2$ in equations (6) and (7) is replaced by the factor α/β . This is consistent with defining wave-breaking to occur when $\sigma = \alpha U_0/N$ for the linear hydrostatic regime. Effectively, this is a simple way of including three-dimensional wave-breaking effects in this calculation. Three-dimensional are not included explcitly in this calculation whereas, by incorporating the anisotropy of the sub-grid orography, they are included in the surface stress calculation[5].

[5]Work is in progress to include three-dimensional wind turning effects explicitly in the stress deposition calculation.

The third and final change to the operational GWD scheme implemented on 6/11/96 was the introduction of another factor, γ^2, into the numerator of equations (13) and (14). Currently γ is set to 0.5 so that it reduces the amplitude of the critical stress vector by a factor of 4. Again, this was motivated by the excessive (and rather violent) GWD seen in the operational model stratosphere. Several middle atmosphere GWD parametrizations have recently utilised such an 'efficiency factor' in their wave-breaking criterion, e.g. Norton and Thuburn (1996). The basic argument put forward for such an efficiency factor is that it accounts for the temporal intermittency of gravity waves; over the course of a model timestep the peak wave momentum flux may be considerably greater than the average.

In the code, equations (13) and (14) require interpolation of the model's prognostic variables from the layer centres to the layer boundaries. This is done with respect to height. Thus,

$$(\Delta z)_{k,k+1/2} = (\Delta \Pi)_{k,k+1/2} \Theta_k C_p / g \qquad \textbf{(15)}$$

is the depth of layer k from its centre to its upper boundary. The layer boundary temperature, $T_{k+1/2}$, is then interpolated from the full level temperatures as

$$T_{k+1/2} = \frac{T_k (\Delta z)_{k+1/2,k+1} + T_{k+1} (\Delta z)_{k,k+1/2}}{(\Delta z)_{k,k+1}} \qquad \textbf{(16)}$$

where $T_k = (\Delta \Pi)_k \Theta_k$. The layer boundary density is calculated as

$$\rho_{k+1/2} = \frac{p_{k+1/2}}{R T_{k+1/2}} \qquad \textbf{(17)}$$

and the static stability as

$$N^2_{k+1/2} = \frac{g(\theta_{k+1} - \theta_k)\,\Pi_{k+1/2}}{T_{K+1/2} (\Delta z)_{k,k+1}} \,. \qquad \textbf{(18)}$$

Note that this is the correct definition of the buoyancy frequency and that it is different to that used in the surface calculations[6].

In the code, the saturation hypothesis is successively applied from the top of level START_L[7] up to the bottom of the top model level. For all levels, apart from START_L and the top one, the stress drop, $\Delta \tau$, equates to a drag on the large scale flow via equation (11). For level START_L, the stress drop is applied evenly over levels 2 to START_L. For the top model level, the saturation hypothesis cannot be applied because there is no obvious way of extrapolating the model prognostic variables

[6] Simple calculations suggest that in extreme conditions, e.g. over high orography in very stable conditions, the differences between the two formulae may be of the order of 10%.

[7] This, in itself, is inconsistent with the surface stress calculation. If the surface layer includes level 4, and this is liable to occur over most of the major orographic regions, then this allows wave-breaking below the level at which the waves were deemed to have been launched upwards.

from the top level to the top of that layer (the model lid). The pragmatic solution employed is to apply an equal drag in the top model level to that applied in the level below, with the proviso that the stress cannot pass through zero. This approach means that some of the stress may 'leak' through the upper boundary of the model, thus effectively implying a reduction in the surface drag.

6. TRAPPED LEE WAVES AND THE REFLECTION OF HYDROSTATIC WAVES

The low-level trapping of short wavelength (less than about 25km) non-hydrostatic lee waves, and the reflection of hydrostatic gravity waves at the tropopause, are both accounted for in the new scheme. Both these mechanisms require the existence of abrupt changes in the vertical profile of the Scorer parameter. The Scorer parameter is defined as

$$l^2 = \frac{N^2}{U^2} - \frac{1}{U}\frac{\delta^2 U}{\delta z^2}\,. \qquad \textbf{(19)}$$

Following Shutts(1990), U is taken to be the component of the wind in the direction of the hydrostatic surface stress. For both the trapped lee wave and transmission coefficient calculations the atmospheric column (up to the current height) is split into halves of equal depth (to be diagnosed). In the following, l_1^2 is the mean Scorer parameter of the bottom layer and l_2^2 is the mean Scorer parameter for the top layer.

The calculation of the trapped lee wave drag involves finding the optimum height, H_l, of the layer interface for the trapped lee wave modes. At this level, their vertical velocity at the ground is zero. As Shutts (1990,equations 22-30) shows, this ultimately involves finding where

$$\frac{\alpha_l \sqrt{l_1^2 - l_2^2} H_l}{\sqrt{1+\alpha_l^2} \tan^{-1}(\alpha_l)} + 1 = 0 \qquad \textbf{(20)}$$

with $\tan(\alpha_l) = \tan^{-1}(m_1 H_l) = -(m_1/m_2)$. Here, m_1 is the vertical wavenumber of the trapped lee waves in the bottom laye, whilst m_2 is the inverse decay scale for the trapped lee waves in the top layer.

The calculation of the vertical transmission coefficient involves splitting the atmospheric column from the ground up to the tropopause into two layers of equal thickness (in terms of height). The transmission coefficient, T, is given by equation (49) of Shutts (1990) as

$$T = \frac{2 l_1 l_2}{l_1^2 + l_2^2 + (l_1^2 - l_2^2)\cos(2 l_1 H_t)} \qquad \textbf{(21)}$$

where H_t is the height above the ground of the interface of the two layers.

In the code, lee waves and partial vertical reflection of hydrostatic waves may only occur when the linear hydrostatic regime is invoked, i.e. when $F_r > 1/\alpha$. The calculation of the Scorer parameter is performed at each model level relevant to either calculation. The Scorer parameter at level k is evaluated as

$$l_k^2 = \frac{N_k^2}{U_k^2} - \frac{(\delta U/\delta z)_{k+1/2} - (\delta U/\delta z)_{k-1/2}}{U_k (\Delta z)_{k+1/2,k-1/2}} \tag{22}$$

where N_k^2 is calculated as in equations (4) and (5) and

$$\left.\frac{\delta U}{\delta z}\right|_{k+1/2} = \frac{U_{k+1} - U_k}{\Delta z_{k,k+1}} . \tag{23}$$

Checks in the code ensure that only realistic values of the Scorer parameter are calculated. These checks[8] terminate the Scorer parameter calculation at level k if $N_k^2 < 0$, or if $U_k < 0.1 ms^{-1}$, $U_k < 0.2 ms^{-1}$ or $U_k < 0.3 ms^{-1}$.

The calculation of the upper and lower layer mean Scorer parameter (l_1^2 and l_2^2 in equations (20) and (21)) is performed as follows. Firstly, for a standard point (i.e. the surface pressure is taken to be 1000hPa), the first level whose top boundary is at least twice the height as the top boundary of the current level is calculated[9]. The Scorer parameter for all model levels from level 2 up to the top of the top layer are then summed together. l_1^2 and l_2^2 are then determined by weighting this sum according to the number of model levels in each of the two layers[10].

Once a level, H_l, which satisfies equation (20) has been found, the trapped lee wave surface stress may be calculated. Consistent with the earlier treatment of the surface layer, the code insists that the interface must be at least the height of the top of level START_L. Additionally, the code only searches for a level where equation (20) is less than zero[11]. Using Shutts (1990, equations 31 and 32), it can be shown that the x-component of the surface stress is given by

$$\tau_{sx_{lee}} = \frac{1}{XY}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} uw\,dx\,dy = \frac{2\alpha_l^2\gamma^3\pi U_s^2 K_* A(K_*,\chi)\,\Delta\chi}{H_l^3(\gamma-1)}\cos\chi \tag{24}$$

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[8] These checks on the wind speed are rather arbitrary and perhaps should be defined in a more rigorous manner.

[9] This is a very dangerous generalisation for an orographic parametrization. Over major orography, a high mean elevation will imply a surface pressure much less than 1000hPa. As a consequence, the positioning of the layer interfaces may be considerably in error.

[10] Clearly, such an averaging process is very crude. By calculating the layer means in this way, rather than some mass weighted sense, the uneveness of the model level thicknesses is not accounted for. As a result of this, and because the calculation always starts at level 2 irrespective of whether that layer of air may be lifted over the mountains, the current diagnosis of trapped lee waves may be somewhat erroneous.

[11] These two factors tend to dominate the positioning of the lee wave interface. With the calculation of the mean Scorer parameter as it is currently, the left hand side of equation (19) is almost always less than zero. Thus, the criterion in the code for the existence of lee waves is almost always satisfied at every level, and thus the height of the interface is almost always level START_L.

where

$$K_* A(K_*, \chi) = \frac{3K_{lee}}{4\pi K_*^{3/2}} [(4\cos^2\chi - 1)\sigma_{xx} + (8\cos\chi\sin\chi)\sigma_{xy} + (4\sin^2\chi - 1)\sigma_{yy}] .$$

(25)

Here,

$$K_{lee} = \frac{1}{K_u^{3/2} - K_l^{3/2}} , \qquad \textbf{(26)}$$

$K_*^2 = \dfrac{l_1^2 + \alpha_1^2 l_2^2}{\alpha_1^2 + 1}$, $\gamma = \dfrac{\tan^{-1}(\alpha_1)}{\alpha_1}$ and $\Delta\chi = 1^0$, the maximum angular width of the lee wave train. The equation for $\tau_{ys_{lee}}$ is as per equation (24), with $\cos\chi$ replaced by $\sin\chi$.

The important scaling parameter for this part of the parametrization is K_{lee}. For the operational model this is currently set at $1.8 \times 10^5 m^{-3/2}$, whilst for the climate model it is set to $3.0 \times 10^5 m^{-3/2}$.

The final stage in the lee wave calculation is to determine the vertical stress profile of the lee waves and hence their drag on the atmosphere. According to Mitchell and Gregory (1994), Shutts (1990, equations 39-41) may be used to determine the proportion of the surface stress remaining at the layer interface. Thus,

$$\frac{\overline{uw}_{z=H_1}}{\overline{uw}_{z=0}} = \frac{\alpha_1^2 l_2^2}{(l_1^2 + \alpha_1^2 l_2^2)(MH_1 + 1)} , \qquad \textbf{(26)}$$

where $M^2 = \dfrac{l_1^2 - l_2^2}{\alpha_1^2 + 1}$.

In the code, the stress profile is calculated assuming that the bottom of the surface layer is the bottom of level 3. Thus, the drag in layers 2 and 3 is less than it should be if the interface of the two lee wave layers is at level 4 or higher[12].

[12] As we have already mentioned the crudeness of the calculation of the layer mean Scorer parameter ensures that the lee wave interface is almost always at the top of level 3.

The calculation of the transmission coefficient (equation (21)) uses l_1^2 and l_2^2 calculated with the interface of the two layers being the first level boundary less than 500hPa. The interface height, H_t, is calculated as the height from the bottom of level 2 up to the interface, using equation (3) to calculate the depth of each level. Note that in the code, the transmission coefficient is not allowed to exceed unity. However, this is perhaps not a valid restriction because the wave reflection may actually enhance the surface stress in some instances (Klemp and Lilly, 1975).

ACKNOWLEDGEMENT.

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7. REFERENCES

Gregory, D., G.J. Shutts and J.R. Mitchell, 1996. A new gravity wave drag scheme incorporating anisotropic orography and low level wave breaking: impact upon the climate of the U.K. Met. Office Unified Model. *Climate Research Technical Note Number 75. Available from the Hadley Centre, Meteorological Office. London Road, Bracknell, Berks, RG12 2SY. Also accepted for publication by Quart.J.R. Meteor. Soc.*

Jones, C.P., 1995. Specification of Ancillary Fields. *Unified Model Documentation Paper 73, available from Unified Model librarian, Meteorological Office, London Road, Bracknell, Bershire, RG12 2SX.*

Klemp, J.B. and D.K. Lilly, 1975. The dynamics of wave-induced downslope winds. *J. Atmos. Sci., 32, 320-339.*

Milton, S.F., and C.A. Wilson, 1996. The impact of parametrized subgrid-scale orographic forcing on systematic errors in a global NWP model. *Mon. Wea. Rev., 124, 2023-2045.*

Mitchell, J.R. and D. Gregory, 1994. A new gravity wave drag parametrization scheme using anisotropic orography, with treatment of flow blocking, high drag jump states, vertical reflection and trapped lee waves. *Unpublished internal Meteorological Office document.*

Norton, W.A. and J. Thuburn, 1996. The two-day wave in a middle atmosphere GCM. *Geophys. Res. Lett.*, 23, 2113-2116.

Palmer, T.N., G.J. Shutts and R. Swinbank, 1986. Alleviation of a systematic westerly bias in general circulation and numerical weather prediction models through an orographic gravity wave drag parametrization. *Quart.J.R. Meteor. Soc., 112,1001-1039.*

Peltier, W.R. and T.L. Clark, 1979. The evolution and stability of finite amplitude mountain waves. Part II: surface wave drag and severe downslope windstorms. *J. Atmos. Sci., 36, 1498-1529.*

Robinson, D., 1994. Orographic Gradient Fields.. *Unpublished internal Meteorological Office Document.*

Rottman, J.W. and R.B. Smith, 1989. A laboratory model of severe downslope winds. *Tellus, 41A, 401-415.*

Shutts, G.J., 1990. A new gravity wave drag parametrization scheme for the unified model. *Turbulence and Diffusion Note 204, available from APR-Division, Meteorological Office, London Road, Bracknell,* *Berkshire, RG12 2SX.*

Smith, R.B., 1985. On severe downslope winds. *J. Atmos. Sci., 42, 2597-2603.*

Smith, R.N.B. and S.D. Jackson, 1997. A parametrization of orographic form drag in the U.K. Meteorological Office Unified Model. *In preparation.*

Wallace, J.M., S. Tibaldi and A.J. Simmons, 1983. Reduction of systematic errors in the ECMWF model through the introduction of envelope topography. *Quart. J.R. Meteor. Soc, 109, 683-717.*